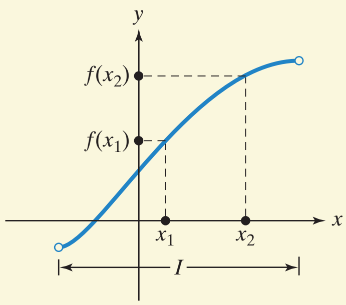
Increasing, Decreasing, and Constant Functions

1. A function is **increasing** on an open interval, I, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.

2. A function is **decreasing** on an open interval, I, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.

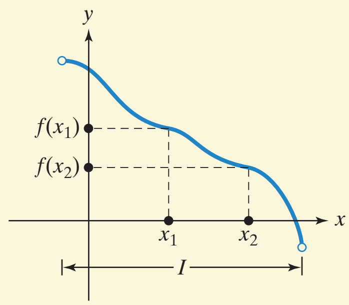
3. A function is **constant** on an open interval, I, if $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval.

Increasing



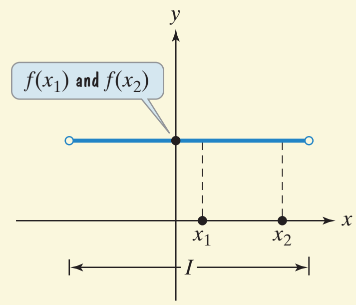
(1) For $x_1 < x_2$ in I,
$f(x_1) < f(x_2)$;
f is increasing on I.

Decreasing



(2) For $x_1 < x_2$ in I,
$f(x_1) > f(x_2)$;
f is decreasing on I.

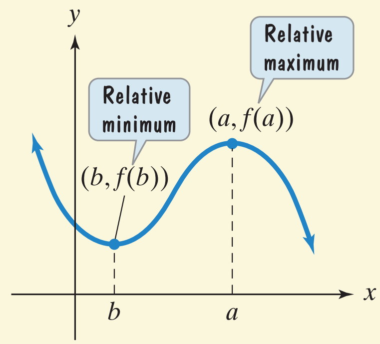
Constant



(3) For x_1 and x_2 in I,
$f(x_1) = f(x_2)$;
f is constant on I.

Definitions of Relative Maximum and Relative Minimum

1. A function value $f(a)$ is a **relative maximum** of f if there exists an open interval containing a such that $f(a) > f(x)$ for all $x \neq a$ in the open interval.
2. A function value $f(b)$ is a **relative minimum** of f if there exists an open interval containing b such that $f(b) < f(x)$ for all $x \neq b$ in the open interval.



The word *local* is sometimes used instead of *relative* when describing maxima or minima.

[9] **4.** Use the graph of the function f given below to find

a) $f(3) =$

b) $f(-3) =$

c) $(f \circ f)(5) =$

d) The domain of f.

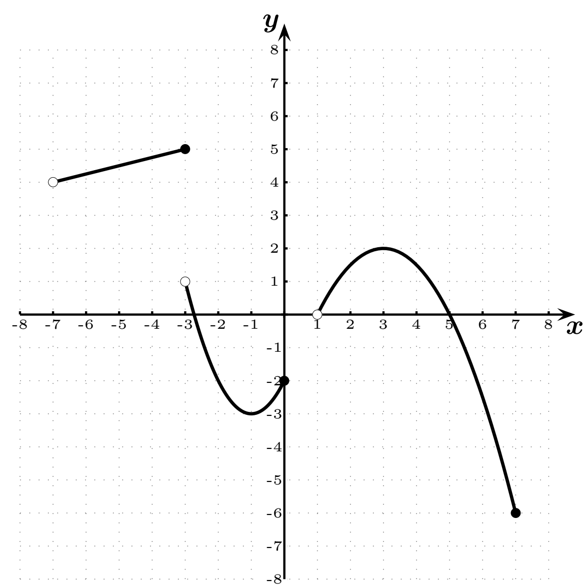
e) The range of f.

f) The interval(s) where f is increasing.

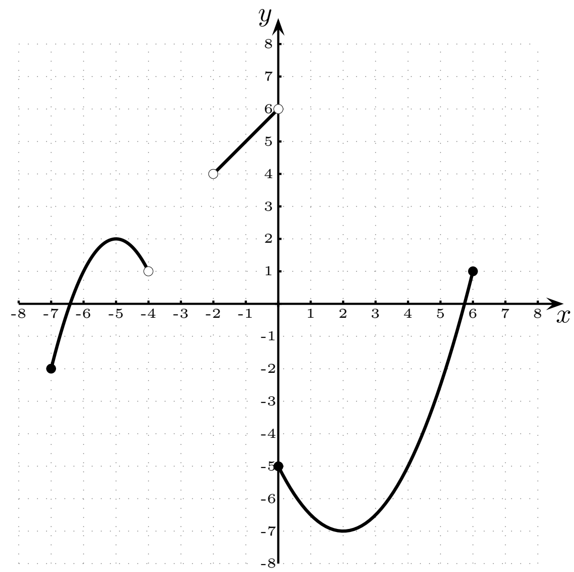
g) The interval(s) where f is decreasing.

h) The relative maxima of f.

i) The relative minima of f.



2. Use the graph of the function f given below to find



a) $f(2) =$

b) $f(-6) =$

c) $(f \circ f)(-5) =$

d) The domain of f.

e) The range of f.

f) The interval(s) where f is increasing.

g) The interval(s) where f is decreasing.

h) The relative maxima of f.

i) The relative minima of f.

Definitions of Even and Odd Functions

The function f is an **even function** if

$$f(-x) = f(x) \quad \text{for all } x \text{ in the domain of } f.$$

The right side of the equation of an even function does not change if x is replaced with $-x$.

The function f is an **odd function** if

$$f(-x) = -f(x) \quad \text{for all } x \text{ in the domain of } f.$$

Every term on the right side of the equation of an odd function changes sign if x is replaced with $-x$.

Even Functions and *y*-Axis Symmetry

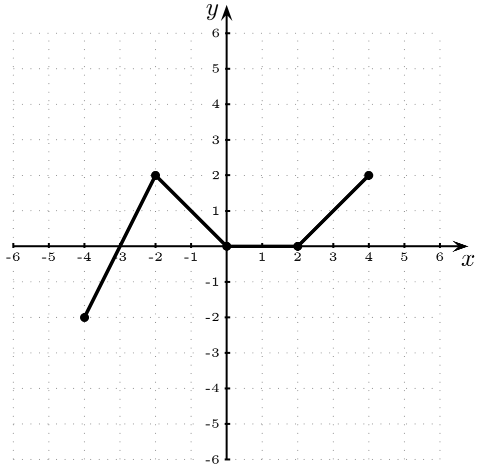
The graph of an even function in which $f(-x) = f(x)$ is symmetric with respect to the y-axis.

Odd Functions and Origin Symmetry

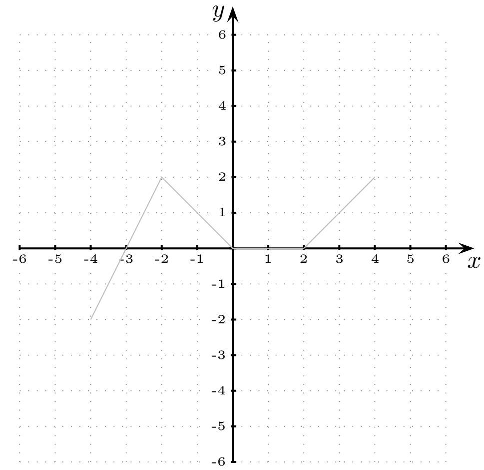
The graph of an odd function in which $f(-x) = -f(x)$ is symmetric with respect to the origin.

[15] **7.** The following is the graph of the function f. In the coordinate axes given below, sketch the graph of the indicated functions. (As a reference, the graph of f is given in each coordinate axes in light gray.)

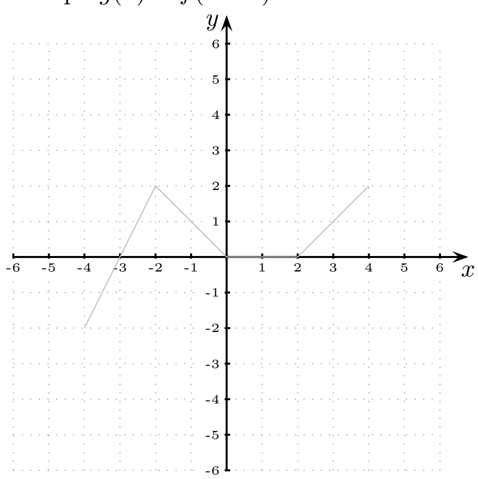
This is the given original graph of f.



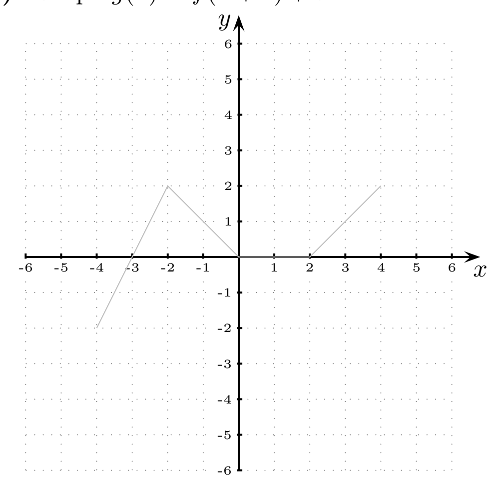
a) Graph $g(x) = f(x) - 3$.



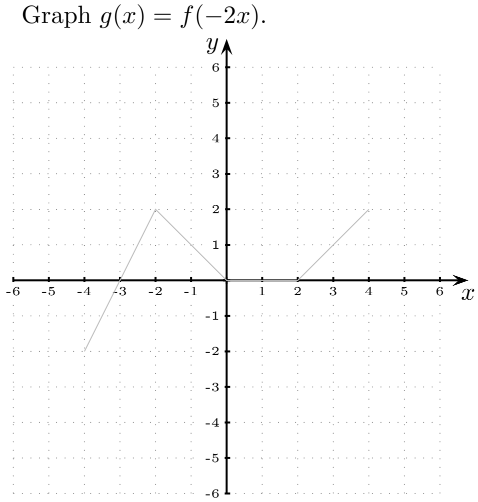
b) Graph $g(x) = f(x - 1)$.



c) Graph $g(x) = f(x + 2) + 3$.



d) Graph $g(x) = f(-2x)$.



e) Graph $g(x) = -2f(x)$.