Final exam review guide. Math 30 (Precalculus). Fall 2023.

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This is a list of topics that you should know well from each section, and which exercises from the book you can do practice that topic.

NOTE: The list of exercises is very long. It does not mean that you have to do all these exercises. Rather, for each topic in each section, try a couple of exercises. If everything is very clear, move on to the next topic; otherwise try a few more exercises and ask for help if you need.

- **Section 1.1**
  - Evaluate functions given their expression. *Ex. 27, 30–33, 35*
  - When a graph is the graph of a function: vertical line test. *Ex. 40–43*
  - Evaluate functions given their graph. *Ex. 53, 54, 56, 58, 59*
  - Evaluate functions given table of values. *Ex. 66, 67*
- **Section 1.2**
  - Understand the concept of domain and range.
  - Finding domain of a function given its expression. *Ex. 6, 8, 10, 14, 15, 18, 25*
  - Finding domain and range of a function given its graph. *Ex. 33–37*
  - Graphing and evaluating piecewise-defined functions. *Ex. 38, 39, 46–48, 51*
- **Section 1.3**
  - Understand what is meant by a function to be increasing or decreasing in an interval.
  - Understand what is meant by a local maximum or minimum, and by an absolute maximum or minimum.
  - Find intervals of increase and decrease, maxima and minima given the graph of a function. *Ex. 18–25.*
- **Section 1.4** (page 1102 only)
  - Understand addition, subtraction, multiplication and division of functions.
  - Understand composition of functions.
  - Find expressions for the combinations or composition of functions (both numerical and with variables). *Ex. 5, 11, 12, 16, 17, 18, 22*
  - Write a function as a composition of two functions. *Ex. 26, 27*
  - Find values of composition of functions given their graphs. *Ex. 42–49*
  - Find values of composition of functions given tables of their values. *Ex. 66-73*
- **Section 1.5**
  - Understand vertical and horizontal translations both in the expression of the function and in their graphs. *Ex. 6, 7, 10–19*
  - Graph vertical and horizontal translations of a given graph given the formula *Ex. 24–26, 28*
  - Find the expression given the graph of a translated function. *Ex. 33–36*
  - Understand reflections, compressions and stretches, both in the expression of the function and in their graphs. *Ex. 53–57*
  - Graph transformed functions using all transformations. *Ex. 71, 74*

- **Section 1.6**
  - Solve absolute value equations. *Ex. 10–15, 19, 21*
  - Solve absolute value inequalities. *Ex. 29–33*
  - Graph absolute value functions. *Ex. 37–41, 45, 46*
- **Section 1.7**
  - Understand the concept of inverse function.
  - Find the inverse of a function given its expression. *Ex. 7–12, 13*
  - Check that two functions are inverses. *Ex. 17, 18*
  - Understand what is a one-to-one function. Horizontal line test. *Ex. 23, 24*
  - Find values of the inverse given graph. *Ex. 25–28*
  - Graph of the inverse. *Ex. 29–32*
  - Find values of the inverse given a table of values. *Ex. 33–41*
- **Section 2.1**
  - Understand concept of linear function.
  - Find slope of line through two points. *Ex. 25–29.*
  - Find equation of line through two points. *Ex. 30–35*
  - Find slope given graph. *Ex. 39–45*
  - Determine if a function is linear given a table of values. *Ex. 47, 47*
- **Section 2.2**
  - Find x and y intercepts of an equation. *Ex. 12–15*
  - Find slopes of lines and determine if they are parallel, perpendicular or neither. *Ex. 18–20*
  - Find equations of parallel and perpendicular lines. *Ex. 24–27*
  - Find slope given graph. *32–37*
  - Sketch the graph of a linear equation. *44–50.*
- **Section 3.2**
  - Understanding quadratic functions: $f(x) = ax^2 + bx + c$.
  - Understanding that the graph of a quadratic function is a parabola.
  - Standard form of a quadratic function. Completing the square. *Ex. 1, 7, 9, 11, 13, 45, 47.*
  - Find vertex, axis of symmetry, y-intercepts and x-intercepts of a quadratic function.
  - Graph quadratic functions. *Ex. 53 to 64 odd numbered, 65.*
  - Find maximum/minimum of a quadratic function *Ex. 15, 17.*

- **Section 3.3**
  - Identifying power functions and polynomial functions. *Ex. 7, 9, 11.*
  - Understand the concept of end behavior of a function. Remember that for polynomials there are four types of end behavior: $\nwarrow\ \nearrow$, $\nwarrow\ \searrow$, $\swarrow\ \nearrow$, and $\swarrow\ \searrow$
  - Find the degree, leading term, leading coefficient, and constant term of a polynomial. *Ex. 1, 2, 12, 13, 15.*
  - Find the end behavior of polynomial functions. *Ex. 17 to 23.*
  - Turning points of polynomial functions: at most, the number of turning points is one less than the degree. *Ex. 4, 31, 33, 35.*
  - Finding y-intercepts of polynomial functions: it is just $f(0)$.
  - Finding x-intercepts of polynomial functions: solve $f(x) = 0$. *Ex. 25, 26, 27, 29.*
  - At most, the number of x-intercepts is the same as the degree.
- **Section 3.4**
  - Knowing that the x-intercepts of a function are also called "zeros" and "roots". They are the solutions of $f(x) = 0$ (note: in the book, x-intercept means a point where the function touches the x-axis, and zero means the x coordinate of that point; we use the terms as synonyms).
  - Solve $f(x) = 0$ by factoring. *Ex. 7 to 23, odd numbered.*
  - Multiplicities of zeros of a polynomial. *Ex. 31 to 41, odd numbered.*
  - Undestand the local behavior at the zeros of a polynomial using the multiplicities.
  - Use the end behavior and the local behavior at the x-intercepts to graph polynomial functions. *Ex. 42 to 47.*
  - Finding a formula of a polynomial function given its graph. *Ex. 41 to 55, odd numbered.*
  - Finding a formula of a polynomial function given some information. *Ex. 57 to 63, odd numbered.*
- **Section 3.5**
  - Long division of polynomials. *Ex. 3, 5, 7, 9, 11.*
  - Dividend, divisor, remainder and quotient of division of polynomials. Division algorithm: $D = d \cdot q + r$.
  - Synthetic division of polynomials. *Ex. 15 to 27, odd numbered, 49, 51, 53.*
- **Section 3.6**
  - Understand the Remainder Theorem: the value $f(a)$ is the same as the remainder we get when we divide $f(x)$ by $(x - a)$. *Ex. 7, 9, 11.*
  - Evaluate polynomials using the remainder theorem.
  - Understand the Factor Theorem: the value a is a zero of $f(x)$ means that $(x - a)$ is a factor of $f(x)$.
  - Finding all the zeros of a polyomial given a factor. *Ex. 15, 17, 19.*
  - The Rational Zero Theorem: if a is a zero of $f(x)$, then it must have the form p/q, where p divides the constant coefficient and q divides the leading coefficient.
  - Use the Rational Zero Theorem to find all the possible rational zeros of a polynomial. *Ex. 2, 23 to 39 odd numbered, 41, 43.*
  - Finding all the zeros of (or factoring) polynomial functions. *Ex.5, 23 to 39 odd numbered, 67, 69.*
  - Fundamental Theorem of Algebra: the number of zeros of a polynomial is exactly the same as its degree (note: some zeros may be complex numbers).
  - Complex conjugate theorem: If $a + bi$ is a zero of a polynomial, then its conjugate $a - bi$ is also a zero.

- **Section 3.7**
  - Identifying rational functions. *Ex. 1, 2, 4, 5.*
  - Domain of rational functions. *Ex. 7, 8.*
  - Vertical asymptotes of rational functions.
  - Horizontal asymptotes of rational functions.
  - Finding horizontal and vertical asymptotes of rational functions. *Ex. 11 to 19, odd numbered.*
  - Graphing rational functions *Ex. 21, 23, 25, 39 to 49 odd numbered.*
  - Writing a rational function given its graph, *Ex. 57, 59, 61.*
- **Section 4.1**
  - Identifying exponential functions: they have the form $f(x) = ab^x$. *Ex. 2, 4.*
  - Evaluating exponential functions. *Ex. 45, 47, 49.*
  - Understanding exponential growth. *Ex. 5, 9, 11, 15, 17.*
  - Compound interest. *Ex. 31 .*
  - Continuous interest. The number e. *Ex. 39, 41.*
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- **Section 4.2**
  - Graphs of exponential functions: horizontal asymptote only on one end. *Ex. 9, 11, 13, 19, 21.*
  - Transformations of graphs of exponential functions. *Ex. 3, 5, 27, 29, 33.*
- **Section 7.6 (polynomial and rational inequalities)**
  - Solving polynomial and rational inequalities. *Ex. 6, 7 from* `https://fsw01.bcc.cuny.edu/luis.fernandez01/web/teaching/classes/math30/oldhw/pdfs/ex07.pdf`
- **Section 4.1**
  - Identifying exponential functions. *Ex. 14, 15, 17.*
  - Understand the formula for compound interest and the exponential function with base e. *Ex. 28, 29, 30, 31.*
  - Evaluate exponential functions. *Ex. 44–49.*
- **Section 4.2**
  - Graph exponential functions. *Ex. 26–28, 32–35.*
- **Section 4.3**
  - Understanding the definition of logarithm. Converting between logarithmic and exponential form of an equation. *Ex. 6–15, 16–25, 26–32, 36–41.*
  - Understand that $\log_b x$ is the inverse of the function b^x.
  - Evaluating logarithms. *Ex. 42–49.*
  - Understand that $\log x$ means $\log_{10} x$ and $\operatorname{Ln} x$ means $\log_e x$.
- **Section 4.4**
  - Graph logarithmic functions. *Ex. 26–33, 34–37, 41–46.*

- **Section 4.5**
  - Understand the product rule, the quotient rule and the power rule for logarithms.
  - Use the rules of logarithms to expand and condense logarithmic expressions. *Ex. 3–24, 27–29, 30–32.*
  - Understand and be able to use the change of base formula. *Ex. 25, 26, 33–37.*
- **Section 4.6**
  - Be able to solve exponential equations. *Ex. 4–10, 11-20, 23, 27.*
  - Be able to solve logarithmic equations. *Ex. 29–43, 44–50.*
- **Section 5.1**
  - Understand the concept of angle, initial side, terminal side.
  - Be able to draw angles in standard position, both when given in degrees and when given in radians. *Ex. 6–31.*
  - Know the values of the special angles, in degrees and in radians.
  - Be able to convert between degrees and radians. *Ex. 26–39.*
  - Finding coterminal angles, both in degrees and in radians. *Ex. 40–53, 54–57.*
  - Finding length of arcs and area of sectors. *Ex. 24, 25, 41, 43.*
- **Section 5.2**
  - Understand the definition of sine and cosine of any angle: the unit circle. *Ex. 6–9, 60–79.*
  - Knowing the Pythagorean identity: $\sin^2\alpha + \cos^2\alpha = 1$.
  - Knowing the sine and the cosine of the special angles, both when given in degrees and when given in radians. *Ex. 10–22.*
  - Finding reference angles, both in degrees and in radians. *Ex. 23–33.*
  - Use reference angles to find the value of sine and cosine of any angle. *Ex. 34–49.*
  - Given the value of the sine or the cosine of an angle, and the quadrant, be able to find the other one. *Ex. 50–53.*
- **Section 5.3**
  - Understand and remember the definitions of tangent, secant, cotangent and cosecant. *Ex. 49–51.*
  - Finding the value of all the trigonometric functions of the special angles. *Ex. 6–31.*
  - Given the value of one of the trigonometric functions, and the quadrant, be able to find the value of all the other ones. *Ex. 38–41.*
- **Section 5.4**
  - Given a right triangle, understand the definition of the trigonometric functions of the acute angles of the triangle as quotients of the sides of the triangle (for example $\sin\alpha$ = opposite/hypotenuse. *Ex. 17–22, 23–28.*
  - Use trigonometry to find the length of one side of a right triangle if one acute angle and the length of another side is given. *Ex. 10–16, 17–22, 29–31.*

- **Section 6.1**
  - Understand the concept of periodic function. Know that $\sin x$, $\cos x$, $\sec x$, and $\csc x$ are periodic with period 2π, and $\tan x$ and $\cot x$ is periodic with period π. *Ex. 1, 2.*
  - Be able to graph $\sin x$ and $\cos x$, appropriately labeling the axes.
  - Find the period, amplitude, and phase shift of a sinusoidal function. *Ex. 18–26.*
  - Be able to graph sinusoidal functions, appropriately labeling the axes. *Ex. 6–17.*
- **Section 6.3**
  - Understand the definition of $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$.
  - Be able to find $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ of some numbers without a calculator, and of any number with a calculator. *Ex. 8–21, 22, 23.*
- **Section 7.1**
  - Understand what is a trigonometric identity. Know the fundamental trigonometric identities (see "summarizing trigonometric identities" in the textbook. *Ex. 5, 7, 13, 15, 17, 19, 21.*
  - Understand what it means to "verify", or "prove" a trigonometric identity.
  - Be able to prove basic trigonometric identities. *Use the Trigonometric Identities Worksheet in the course webpage:* `https://fsw01.bcc.cuny.edu/luis.fernandez01/web/teaching/classes/math30/hw/trigidentities.pdf`
- **Section 7.2**
  - Solve simple trigonometric equations when the variable lies in an interval. *Ex. 4–12.*
  - Use algebra to solve trigonometric equations. *Ex. 13–22*
  - Solve trigonometric equations involving several trigonometric functions. *Ex. 23–26.*