**MATH 30 - Precalculus. Review exercises.** Professor Luis Fernández

## SOLUTION

SOLUTIONS. Note: only the solution to the even numbered exercises is given. The solution of the odd numbered ones is in the back of the textbook.

**1.** Do exercises **77 and 80** from **Section 1.2** in the book.
**80: a.** Domain: $(-\infty, \infty)$. **b.** Range: $[0, \infty)$. **c.** $x$-intercept: $-1$. **d.** $y$-intercept: 1. **e.** $f(-4) = 3$; $f(3) = 4$.

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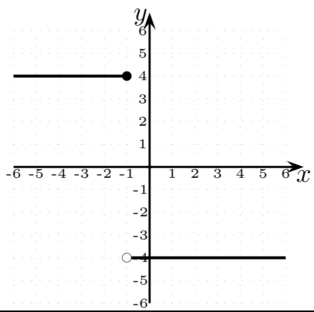
**2.** Do exercises **5, 6, 47 and 50** from **Section 1.3** in the book.
**6: a.** Increasing in $[-3, 2]$. **b.** Never decreasing. **c.** Never constant.
**50: a.**



**b.** Range: the values 4 and $-4$ (in set notation, $\{-4, 4\}$).

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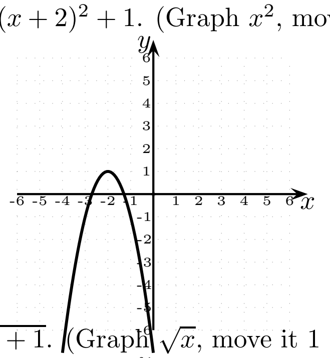
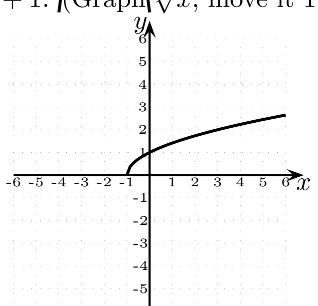
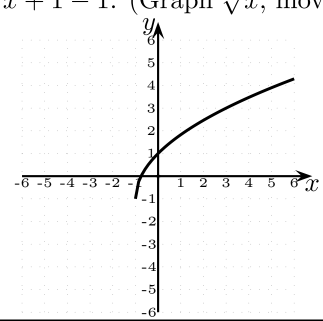
**3.** Do exercises **66, 70 and 80** from **Section 1.6** in the book.
**66:** $h(x) = -2(x+2)^2 + 1$. (Graph $x^2$, move it 2 left, stretch vertically by 2, reflect w.r.t. $x$-axis, then 1 up.)



**70:** $h(x) = \sqrt{x+1}$. (Graph $\sqrt{x}$, move it 1 left.)



**80:** $h(x) = 2\sqrt{x+1} - 1$. (Graph $\sqrt{x}$, move it 1 left, stretch vertically by 2, move it 1 down.)



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**4.** Do exercises **67, 68 and 70** from **Section 1.7** in the book.
**68.** $f(x) = \dfrac{x}{x+5}$, $g(x) = \dfrac{6}{x}$.

**a.** $(f \circ g)(x) = f(g(x)) = \dfrac{\frac{6}{x}}{\frac{6}{x}+5} = \dfrac{6}{5x+6}$

**b.** To find the domain of $f \circ g$, exclude those values of $x$ not in the domain of $g$ and those values of $x$ such that $g(x)$ is not in the domain of $f$.

Domain of $g$: every number excluding 0. Domain of $f$: every number excluding $-5$. For $g(x)$ not to be in the domain of $f$ we solve $g(x) = -5$, so $\frac{6}{x} = -5$, so $x = -\frac{6}{5}$. Therefore the domain of $f \circ g$ is $(-\infty, -\frac{6}{5}) \cup (-\frac{6}{5}, 0) \cup (0, \infty)$.

**70.** $f(x) = \sqrt{x}$, $g(x) = x - 3$.

**a.** $(f \circ g)(x) = f(g(x)) = \sqrt{x-3}$

**b.** To find the domain of $f \circ g$, exclude those values of $x$ not in the domain of $g$ and those values of $x$ such that $g(x)$ is not in the domain of $f$.

Domain of $g$: every number (nothing excluded). Domain of $f$: every nonnegative number, so exclude all the values for which $g(x) < 0$, so $x - 3 < 0$ so $x < 3$. Therefore the domain of $f \circ g$ is $[3, \infty)$.

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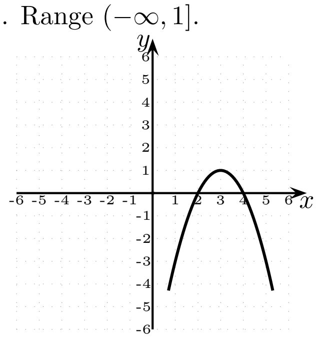
**5.** Do exercise **26** from **Section 2.2** in the book.

**26.** $f(x) = 1 - (x-3)^2$. Vertex: $(3, 1)$. Axis of symmetry: $x = 3$. $x$-intercepts: 4 and 2. $y$-intercepts: $-8$. Domain: $(-\infty, \infty)$. Range $(-\infty, 1]$.

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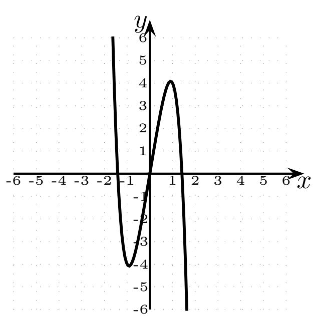
**6.** Do exercises **41, 47, 52** from **Section 2.3** in the book.

**52.** $f(x) = 6x - x^3 - x^5$. **a.** End behaviour: like $-x^5$ (up left and down right). **b.** $x$-intercepts: factor $f(x) = -x(x^4 + x^2 - 6) = -x(x^2-2)(x^2+3) = -x(x+\sqrt{2})(x-\sqrt{2})(x^2+3)$, so they are 0, $\sqrt{2}$ and $-\sqrt{2}$ all with multiplicity 1. **c.** $y$-intercept: $f(0) = 0$. **d.** $f(-x) = 6(-x) - (-x)^3 - (-x)^5 = -6x + x^3 + x^5 = -(6x - x^3 - x^5) = -f(x)$, so $f$ is odd. **e.**

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**7.** Do exercises **11, 13, 33 and 34** from **Section 2.4** in the book.

**34.** $f(3) = -27$.

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**8.** Do exercises **2, 22, 23 and 26** from **Section 2.5** in the book.

**2.** Possible rational roots of $x^3 + 3x^2 - 6x - 8$ are: $\pm1, \pm2, \pm4, \pm8$.

**22.** $2x^3 - 5x^2 - 6x + 4 = 0$. **a.** Possible rational roots are: $\pm1, \pm2, \pm4, \pm\frac{1}{2}$. **b.** $\frac{1}{2}$ works, and $2x^3 - 5x^2 - 6x + 4 = 2(x - \frac{1}{2})(x^2 - 2x - 4) = 0$. So one solution is $x = \frac{1}{2}$. **c.** We find the other two by solving $(x^2 - 2x - 4) = 0$ using the quadratic formula. This gives $x = 1 + \sqrt{5}$ and $x = 1 - \sqrt{5}$.

**26.** If $2i$ is a zero, $-2i$ must also be a zero, and $f$ has degree 3, so $f(x) = A(x-4)(x-2i)(x+2i)$. Now $f(-1) = 50$, so $f(-1) = A(-1-4)(-1-2i)(-1+2i) = -25A = 50$, so $A = -2$ and $f(x) = -2(x-4)(x-2i)(x+2i)$.

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**9.** Do exercises **51, 53, and 64** from **Section 2.6** in the book.

**64.** $f(x) = \dfrac{x-4}{x^2-x-6}$.

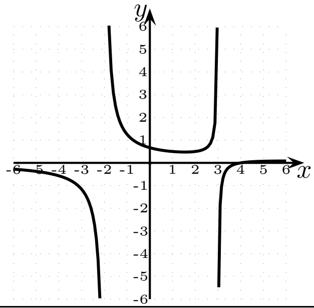
$y$-intercept: $f(0) = \frac{2}{3}$.

$x$-intercept: $x = 4$.

$f$ cannot be even or odd since the $x$-intercepts are not symmetric about 0 (4 is an $x$-intercept but $-4$ is not).

Horizontal Asymptotes: $f(x) \approx \frac{x}{x^2} = \frac{1}{x}$, so $f$ has a horizontal asymptote at $y = 0$.

Vertical Asymptotes: The denominator of $f(x)$ is $(x-3)(x+2)$, so $f$ has vertical asymptotes at 3 and $-2$.



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**10.** Do exercises **11, 16, 53 and 56** from **Section 2.7** in the book.

**16.** $3x^2 + 16x < -5 \Rightarrow 3x^2 + 16x + 5 < 0 \Rightarrow (x+5)(3x+1) < 0$. Do a table of values. The solution is $(-5, -\frac{1}{3})$.

**56.** $\dfrac{x}{x-1} > 2 \Rightarrow \dfrac{x}{x-1} - 2 > 0 \Rightarrow \dfrac{-x+2}{x-1} > 0$. Do a table of values. The solution is $(1, 2)$.

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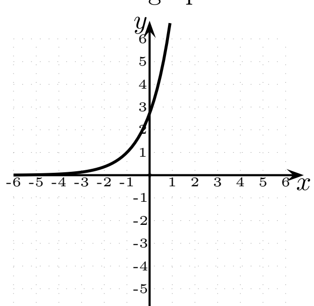
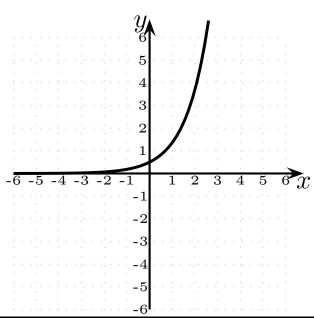
**11.** Do exercises **36 and 44** from **Section 3.1** in the book.

**36.** $f(x) = e^{x+1}$. Move the graph of $e^x$ one unit to the left.



**44.** $f(x) = \frac{1}{2}e^x$. Shrink the graph of $e^x$ vertically by 2.



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**12.** Do exercises **30, 32, 34, 76, 78, 86 and 90** from **Section 3.2** in the book.

**30.** $\log_6 \sqrt{6} = \frac{1}{2}$. **32.** $\log_3 \dfrac{1}{\sqrt{3}} = -\frac{1}{2}$. **34.** $\log_{81} 9 = \frac{1}{2}$.

**76.** Find the domain of $f(x) = \log_5(x+6)$. For something to be in the domain of $\log_b$, that something has to be positive. Therefore we need $x + 6 > 0$, so $x > -6$, so the domain of $f$ is $(-6, \infty)$.

**78.** Find the domain of $f(x) = \log(7-x)$. Again, for something to be in the domain of $\log_b$, that something has to be positive. Therefore we need $7 - x > 0$, so $x < 7$, so the domain of $f$ is $(-\infty, 7)$.

**86.** $10^{\log 53} = 53$. **90.** $\ln e^7 = 7$.

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**13.** Do exercises **40, 58, 62, 72 and 74** from **Section 3.3** in the book.

**40.** $\log\left[\dfrac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right] = \log 100 + 3\log x + \frac{1}{3}\log(5-x) - \log 3 - 2\log(x+7) = 2 + 3\log x + \frac{1}{3}\log(5-x) - \log 3 - 2\log(x+7)$

**58.** $2\ln x - \frac{1}{2}\ln y = \ln\left(\dfrac{x^2}{\sqrt{y}}\right)$.

**62.** $4\ln x + 7\ln y - 3\ln z = \ln\left(\dfrac{x^4y^7}{z^3}\right)$.

**72.** $\log_6 17 = \dfrac{\log 17}{\log 6} = 1.5813$.

**74.** $\log_{16} 57.2 = \dfrac{\log 57.2}{\log 16} = 1.4595$.

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**14.** Do exercises **25, 28, 40, 48** from **Section 4.2** in the book.

**28.** If $\sin t = \frac{2}{3}$ and $\cos t = \frac{\sqrt{5}}{3}$ then $\tan t = \frac{\sin t}{\cos t} = \frac{2}{\sqrt{5}}$, $\sec t = \frac{1}{\cos t} = \frac{3}{\sqrt{5}}$, $\csc t = \frac{1}{\sin t} = \frac{3}{2}$, $\cot t = \frac{\cos t}{\sin t} = \frac{\sqrt{5}}{2}$

**40.** $\csc\frac{9\pi}{4} = \dfrac{1}{\sin\dfrac{9\pi}{4}} = \dfrac{1}{\sin\dfrac{\pi}{4}} = \dfrac{1}{\dfrac{\sqrt{2}}{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$.

**48.** $-\cot\left(\frac{\pi}{4} + 17\pi\right) = -\cot\left(\frac{\pi}{4} + \pi\right) = -\cot\left(\frac{5\pi}{4}\right) = -\dfrac{\cos\left(\dfrac{5\pi}{4}\right)}{\sin\left(\dfrac{5\pi}{4}\right)} = -\dfrac{-\dfrac{\sqrt{2}}{2}}{-\dfrac{\sqrt{2}}{2}} = -1$ (the first equality comes from the fact that $\left(\frac{\pi}{4} + 17\pi\right)$ and $\left(\frac{\pi}{4} + \pi\right)$ are coterminal angles).

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**15.** Do exercises **10, 12, 14, 44 and 54** from **Section 4.3** in the book.

**10.** $\tan 30° = \dfrac{1}{\sqrt{3}}$.

**12.** $\csc 45° = \dfrac{\sqrt{2}}{1} = \sqrt{2}$.

**14.** $\cot\frac{\pi}{3} = \cot 60° = \dfrac{1}{\sqrt{3}}$.

**44.** $\dfrac{1}{\cot\dfrac{\pi}{4}} - \dfrac{2}{\csc\dfrac{\pi}{6}} = \dfrac{\sin\dfrac{\pi}{4}}{\cos\dfrac{\pi}{4}} - 2\sin\frac{\pi}{6} = \dfrac{\dfrac{\sqrt{2}}{2}}{\dfrac{\sqrt{2}}{2}} - 2\cdot\frac{1}{2} = 1 - 1 = 0$.

**54.** By looking at the picture, $\tan 40° = \dfrac{h}{35}$, so $h = 35\tan 40° \approx 29ft$.

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**16.** Do exercises **25, 26, 68, 72 and 74** from **Section 4.4** in the book.

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**26.** Suppose $\cos\theta = \frac{4}{5}$. Draw a right triangle whose leg adjacent to $\theta$ has length 4 and whose hypotenuse has length 5. Then the leg opposite to $\theta$ has lenght $\sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$ by the Pythagorean theorem. So $\sin\theta = \pm\frac{3}{5}$. Since $\theta$ is in quadrant IV, $\sin\theta$ is negative, so $\sin\theta = -\frac{3}{5}$ and from here $\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{3}{4}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$, $\csc\theta = \frac{1}{\sin\theta} = -\frac{5}{3}$, and $\cot\theta = \frac{\cos\theta}{\sin\theta} = -\frac{4}{3}$.

**68.** $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$.

**72.** $\tan\frac{9\pi}{2} = \tan\frac{8\pi+\pi}{2} = \tan\left(\frac{8\pi}{2} + \frac{\pi}{2}\right) = \tan\left(4\pi + \frac{\pi}{2}\right) = \tan\frac{\pi}{2}$, which is undefined.

**74.** $\sin(-225°) = \sin(-225° + 360°) = \sin(135°) = \sqrt{2}2$.

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