Practice Exercises for Logarithmic Equations

1. Solve the logarithmic equation: $\log_2(x) = 5$.
2. Solve the logarithmic equation: $\log_3(x - 2) = 4$.
3. Solve the logarithmic equation: $\ln(x) = 2$.
4. Solve the logarithmic equation: $\log(x + 3) - \log(x) = 1$.
5. Solve the logarithmic equation: $\log_5(x^2) = 4$.
6. Solve the logarithmic equation: $\log_2(2x + 1) = 3$.
7. Solve the logarithmic equation: $\log_7(x + 1) = 2$.
8. Solve the logarithmic equation: $\ln(3x) = \ln(5) + \ln(x)$.
9. Solve the logarithmic equation: $\log_3(2x + 1) = 2$.
10. Solve the logarithmic equation: $\log_4(x + 5) - \log_4(x) = 1$.
11. Solve the logarithmic equation: $2\log(x) = 3$.
12. Solve the logarithmic equation: $\log_2(x + 5) = \log_2(2x)$.
13. Solve the logarithmic equation: $\log_5(2x) = 3$.
14. Solve the logarithmic equation: $\ln(x^2) = 4$.
15. Solve the logarithmic equation: $\log(x + 4) = \log(x + 2) + \log(2)$.

Solutions

1. Solve the logarithmic equation: $\log_2(x) = 5$.

   Solution:
   $$\log_2(x) = 5 \quad \Rightarrow \quad x = 2^5 = 32.$$

2. Solve the logarithmic equation: $\log_3(x-2) = 4$.

   Solution:
   $$\log_3(x-2) = 4 \quad \Rightarrow \quad x - 2 = 3^4 = 81 \quad \Rightarrow \quad x = 83.$$

3. Solve the logarithmic equation: $\ln(x) = 2$.

   Solution:
   $$\ln(x) = 2 \quad \Rightarrow \quad x = e^2 \approx 7.389.$$

4. Solve the logarithmic equation: $\log(x+3) - \log(x) = 1$.

   Solution: Using the logarithmic property $\log(a) - \log(b) = \log\left(\frac{a}{b}\right)$, we get:
   $$\log\left(\frac{x+3}{x}\right) = 1 \quad \Rightarrow \quad \frac{x+3}{x} = 10 \quad \Rightarrow \quad x + 3 = 10x \quad \Rightarrow \quad 3 = 9x \quad \Rightarrow \quad x = \frac{1}{3}.$$

5. Solve the logarithmic equation: $\log_5(x^2) = 4$.

   Solution: Using the logarithmic property $\log_b(a^n) = n\log_b(a)$, we have:
   $$2\log_5(x) = 4 \quad \Rightarrow \quad \log_5(x) = 2 \quad \Rightarrow \quad x = 5^2 = 25.$$

6. Solve the logarithmic equation: $\log_2(2x+1) = 3$.

   Solution:
   $$\log_2(2x+1) = 3 \quad \Rightarrow \quad 2x + 1 = 2^3 = 8 \quad \Rightarrow \quad 2x = 7 \quad \Rightarrow \quad x = \frac{7}{2}.$$

7. Solve the logarithmic equation: $\log_7(x+1) = 2$.

   Solution:
   $$\log_7(x+1) = 2 \quad \Rightarrow \quad x + 1 = 7^2 = 49 \quad \Rightarrow \quad x = 48.$$

8. Solve the logarithmic equation: $\ln(3x) = \ln(5) + \ln(x)$.

   Solution: Using the logarithmic property $\ln(a) + \ln(b) = \ln(ab)$, we get:
   $$\ln(3x) = \ln(5x) \quad \Rightarrow \quad 3x = 5x \quad \Rightarrow \quad -2x = 0 \quad \Rightarrow \quad x = 0.$$

   But $x = 0$ is not a valid solution, since logarithms are undefined at 0.

9. Solve the logarithmic equation: $\log_3(2x+1) = 2$.

   Solution:
   $$\log_3(2x+1) = 2 \quad \Rightarrow \quad 2x + 1 = 3^2 = 9 \quad \Rightarrow \quad 2x = 8 \quad \Rightarrow \quad x = 4.$$

10. Solve the logarithmic equation: $\log_4(x+5) - \log_4(x) = 1$.

    Solution: Using the logarithmic property $\log_b(a) - \log_b(b) = \log_b\left(\frac{a}{b}\right)$, we get:
    $$\log_4\left(\frac{x+5}{x}\right) = 1 \quad \Rightarrow \quad \frac{x+5}{x} = 4^1 = 4 \quad \Rightarrow \quad x + 5 = 4x \quad \Rightarrow \quad 5 = 3x \quad \Rightarrow \quad x = \frac{5}{3}.$$

11. Solve the logarithmic equation: $2\log(x) = 3$.

    Solution:
    $$2\log(x) = 3 \quad \Rightarrow \quad \log(x) = \frac{3}{2} \quad \Rightarrow \quad x = 10^{\frac{3}{2}} = \sqrt{1000} \approx 31.62.$$

12. Solve the logarithmic equation: $\log_2(x+5) = \log_2(2x)$.

    Solution: Since the logarithms have the same base, we can equate the arguments:
    $$x + 5 = 2x \quad \Rightarrow \quad 5 = x \quad \Rightarrow \quad x = 5.$$

13. Solve the logarithmic equation: $\log_5(2x) = 3$.

    Solution:
    $$\log_5(2x) = 3 \quad \Rightarrow \quad 2x = 5^3 = 125 \quad \Rightarrow \quad x = \frac{125}{2} = 62.5.$$

14. Solve the logarithmic equation: $\ln(x^2) = 4$.

    Solution: Using the logarithmic property $\ln(a^n) = n\ln(a)$, we get:
    $$2\ln(x) = 4 \quad \Rightarrow \quad \ln(x) = 2 \quad \Rightarrow \quad x = e^2 \approx 7.389.$$

15. Solve the logarithmic equation: $\log(x+4) = \log(x+2) + \log(2)$.

    Solution: Using the logarithmic property $\log(a) + \log(b) = \log(ab)$, we get:
    $$\log(x+4) = \log(2(x+2)) \quad \Rightarrow \quad x + 4 = 2(x+2) \quad \Rightarrow \quad x + 4 = 2x + 4 \quad \Rightarrow \quad x = 0.$$

    However, $x = 0$ is not a valid solution because logarithms are undefined at $x = 0$.