Practice Exercises for Exponential Equations

Solve the following exponential equations.

1. $2^{x+1} = 16$.
2. $5^x = 125$.
3. $2^{3x+1} = 16^{x+2}$.
4. $3^{x-1} = 27$.
5. $4^x = 64$.
6. $5^{x+1} = 125^{2x-3}$.
7. $10^{x+1} = 1000$.
8. $9^{x+2} = 27^{x+1}$.
9. $e^{2x} = 7$.
10. $2^x + 3^x = 5^x$.
11. $3^{2x} = 81$.
12. $6^x = 36$.
13. $2^{2x} \cdot 8^{x+1} = 32^{x-1}$.
14. $10^x \cdot 5^x = 25^{x+1}$.
15. $7^{2x} = 49^{x+1}$.

Solutions

1. $2^{x+1} = 16$.

   Solution:
   $$2^{x+1} = 16 \quad \Rightarrow \quad 2^{x+1} = 2^4 \quad \Rightarrow \quad x+1 = 4 \quad \Rightarrow \quad x = 3.$$

2. $5^x = 125$.

   Solution:
   $$5^x = 125 \quad \Rightarrow \quad 5^x = 5^3 \quad \Rightarrow \quad x = 3.$$

3. $2^{3x+1} = 16^{x+2}$.

   Solution:
   $$2^{3x+1} = 16^{x+2} \quad \Rightarrow \quad 2^{3x+1} = (2^4)^{x+2} = 2^{4(x+2)} = 2^{4x+8}.$$
   $$2^{3x+1} = 2^{4x+8} \quad \Rightarrow \quad 3x+1 = 4x+8 \quad \Rightarrow \quad -x = 7 \quad \Rightarrow \quad x = -7.$$

4. $3^{x-1} = 27$.

   Solution:
   $$3^{x-1} = 27 \quad \Rightarrow \quad 3^{x-1} = 3^3 \quad \Rightarrow \quad x-1 = 3 \quad \Rightarrow \quad x = 4.$$

5. $4^x = 64$.

   Solution:
   $$4^x = 64 \quad \Rightarrow \quad (2^2)^x = 2^6 \quad \Rightarrow \quad 2^{2x} = 2^6 \quad \Rightarrow \quad 2x = 6 \quad \Rightarrow \quad x = 3.$$

6. $5^{x+1} = 125^{2x-3}$.

   Solution:
   $$5^{x+1} = 125^{2x-3} \quad \Rightarrow \quad 5^{x+1} = (5^3)^{2x-3} = 5^{3(2x-3)} = 5^{6x-9}.$$
   $$5^{x+1} = 5^{6x-9} \quad \Rightarrow \quad x+1 = 6x-9 \quad \Rightarrow \quad -5x = -10 \quad \Rightarrow \quad x = 2.$$

7. $10^{x+1} = 1000$.

   Solution:
   $$10^{x+1} = 1000 \quad \Rightarrow \quad 10^{x+1} = 10^3 \quad \Rightarrow \quad x+1 = 3 \quad \Rightarrow \quad x = 2.$$

8. $9^{x+2} = 27^{x+1}$.

   Solution:
   $$9^{x+2} = 27^{x+1} \quad \Rightarrow \quad (3^2)^{x+2} = (3^3)^{x+1} \quad \Rightarrow \quad 3^{2(x+2)} = 3^{3(x+1)}.$$
   $$3^{2(x+2)} = 3^{3(x+1)} \quad \Rightarrow \quad 2(x+2) = 3(x+1) \quad \Rightarrow \quad 2x+4 = 3x+3 \quad \Rightarrow \quad x = 1.$$

9. $e^{2x} = 7$.

   Solution:
   $$e^{2x} = 7 \quad \Rightarrow \quad 2x = \ln(7) \quad \Rightarrow \quad x = \frac{\ln(7)}{2}.$$

10. $2^x + 3^x = 5^x$.

    Solution: This equation is difficult to solve algebraically. We can try a numerical or graphical approach. For example, by plotting the left-hand side and right-hand side functions, we find that the solution is approximately $x \approx 1$.

11. $3^{2x} = 81$.

    Solution:
    $$3^{2x} = 81 \quad \Rightarrow \quad 3^{2x} = 3^4 \quad \Rightarrow \quad 2x = 4 \quad \Rightarrow \quad x = 2.$$

12. $6^x = 36$.

Solution:

$$6^x = 36 \quad \Rightarrow \quad 6^x = 6^2 \quad \Rightarrow \quad x = 2.$$

13. $2^{2x} \cdot 8^{x+1} = 32^{x-1}$.

Solution:

$$2^{2x} \cdot 8^{x+1} = 32^{x-1} \quad \Rightarrow \quad 2^{2x} \cdot (2^3)^{x+1} = (2^5)^{x-1}.$$

$$2^{2x} \cdot 2^{3(x+1)} = 2^{5(x-1)} \quad \Rightarrow \quad 2^{2x+3x+3} = 2^{5x-5}.$$

$$2^{5x+3} = 2^{5x-5} \quad \Rightarrow \quad 5x + 3 = 5x - 5 \quad \Rightarrow \quad 3 = -5 \quad \text{(no solution)}.$$

14. $10^x \cdot 5^x = 25^{x+1}$.

Solution:

$$10^x \cdot 5^x = 25^{x+1} \quad \Rightarrow \quad (2 \cdot 5)^x \cdot 5^x = (5^2)^{x+1}.$$

$$2^x \cdot 5^{2x} = 5^{2x+2} \quad \Rightarrow \quad 2^x = 5^2 \quad \Rightarrow \quad 2^x = 25.$$

Taking the logarithm:

$$x \log(2) = \log(25) \quad \Rightarrow \quad x = \frac{\log(25)}{\log(2)} \approx 4.64.$$

15. $7^{2x} = 49^{x+1}$.

Solution:

$$7^{2x} = 49^{x+1} \quad \Rightarrow \quad 7^{2x} = (7^2)^{x+1} \quad \Rightarrow \quad 7^{2x} = 7^{2(x+1)}.$$

$$2x = 2(x+1) \quad \Rightarrow \quad 2x = 2x + 2 \quad \Rightarrow \quad 0 = 2 \quad \text{(no solution)}.$$