MATH 30 - Precalculus. Homework 7. Due Th. 04/11/2019. Professor Luis Fernández

SOLUTION

1. Use a calculator to approximate the following numbers to 4 decimal places.

a) $2^{3.4} = 10.5560$ **b)** $e^{1.5} = 4.4817$ **c)** $6^{-\frac{1}{3}} = 1.8171$ **d)** $\sqrt{3}^{\sqrt{2}} = 2.1746$

e) $\log 12 = 1.0792$ **f)** $\log \sqrt{5} = 0.3495$ **g)** $\ln \frac{1}{5} = 1.6094$ **h)** $\ln 469993 = 13.0605$

---

2. Find without using a calculator.

a) $\log_2 8 = 3$ **b)** $\log_3 \frac{1}{3} = -1$ **c)** $\log_6 \sqrt{6} = \frac{1}{2}$ **d)** $\log_{102} 102^4 = 4$

e) $\log_8 2 = \frac{1}{3}$ **f)** $\log_{27} \frac{1}{3} = -\frac{1}{3}$ **g)** $\log_5 1 = 0$ **h)** $\log_3(\log_8 2) = \log_3(\frac{1}{3}) = -1$

---

3. Simplify each expression. Here a is a positive number.

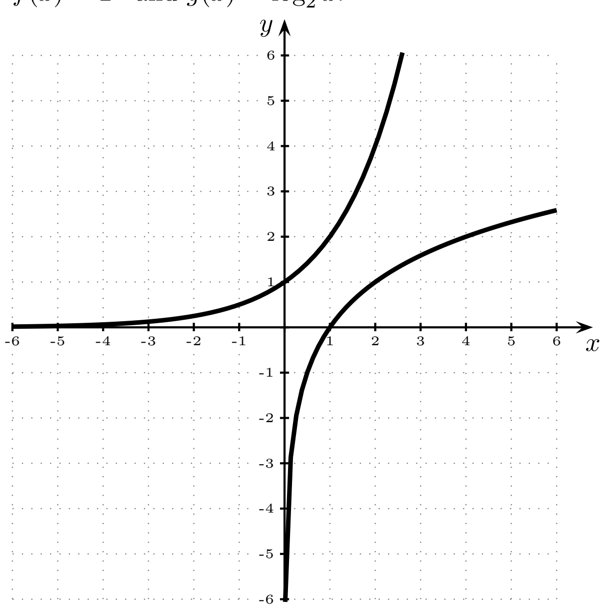
a) $\log_a a^4 = 4$ **b)** $\log_a \frac{1}{a^7} = -7$ **c)** $\log_a a^{\frac{1}{5}} = \frac{1}{5}$ **d)** $\log_a \sqrt[3]{a} = \frac{1}{3}$

e) $2^{\log_2 7} = 7$ **f)** $a^{\log_a \frac{1}{5}} = \frac{1}{5}$ **g)** $10^{\log \sqrt{4}} = \sqrt{4}$ **h)** $e^{\ln 3x^2} = 3x^2$

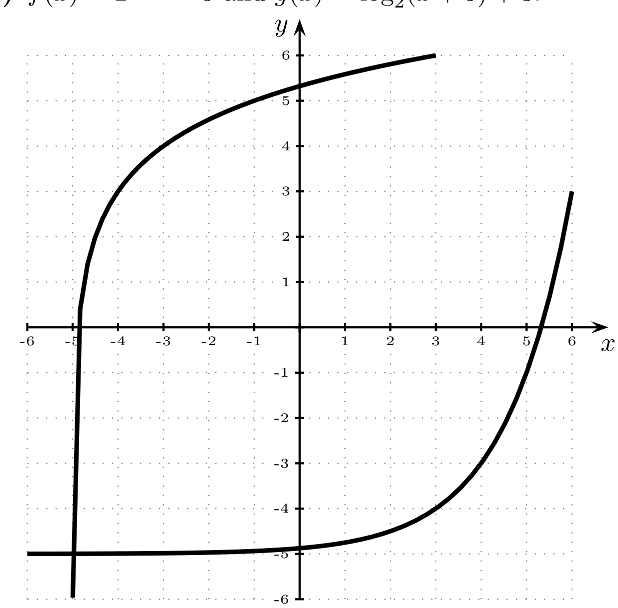
---

4. Graph the following functions in the axes provided (both in the same axes).

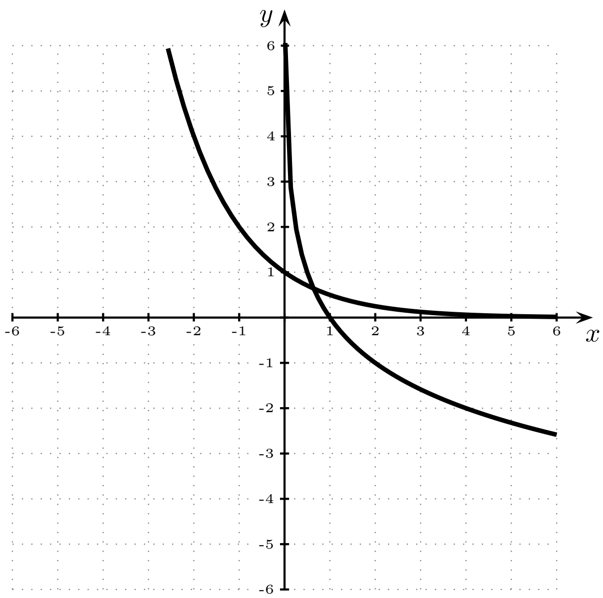
a) $f(x) = 2^x$ and $g(x) = \log_2 x$.



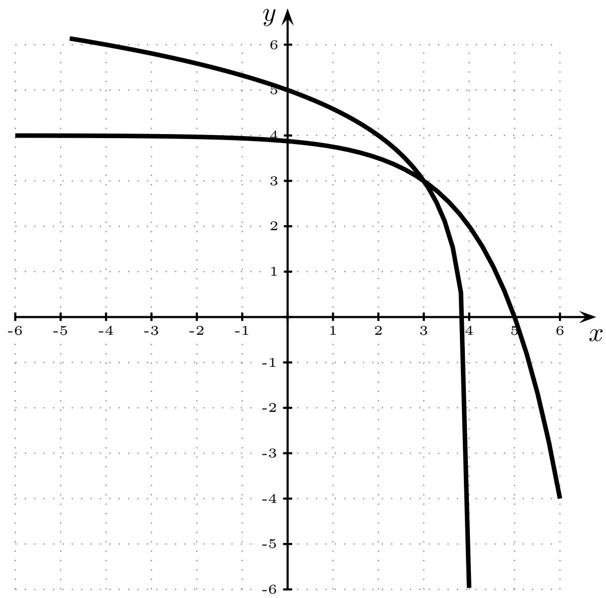
b) $f(x) = 2^{x-3} - 5$ and $g(x) = \log_2(x+5) + 3$.



c) $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \log_{\frac{1}{2}} x$.



d) $f(x) = 4 - 2^{x-3}$ and $g(x) = \log_2(4 - x) + 3$.



---

5. Find the domain of the following logarithmic functions.

a) $f(x) = \log_4(x - 5)$ **b)** $g(x) = \ln(x + 5)^2$ **c)** $h(x) = \ln\left(\dfrac{x-2}{x+1}\right)$

Solutions:

a) $\log_4(x - 5)$ is defined only when $x - 5 > 0$, i.e. when $x > 5$. Therefore the domain of f is $(5, \infty)$.

b) $\ln(x + 5)^2$ is defined only when $(x + 5)^2 > 0$. $(x + 5)^2$ is always positive except at $x = -5$, where it is 0. Therefore -5 is not in the domain, but everything else is, and therefore the domain of g is $(-\infty, -5) \cup (-5, \infty)$.

c) $\ln\left(\frac{x-2}{x+1}\right)$ is defined only when $\frac{x-2}{x+1} > 0$, so we have to solve the inequality $\frac{x-2}{x+1} > 0$. Do a table of signs as in the previous exercises to find that the solution of this inequality is $(-\infty, -1) \cup (2, \infty)$. Therefore the domain of h is $(-\infty, -1) \cup (2, \infty)$.

---

6. Find the inverse of the following functions.

a) $f(x) = 4e^{x+2} - 3$ **b)** $g(x) = 2 + \log_4(2x - 3)$

Solutions:

a) We have to solve for y in the equation $f(y) = x$, or $4e^{y+2} - 3 = x$

$$4e^{y+2} - 3 = x \Rightarrow 4e^{y+2} = x + 3 \Rightarrow e^{y+2} = \frac{x+3}{4} \Rightarrow y + 2 = \ln\left(\frac{x+3}{4}\right) \Rightarrow y = \ln\left(\frac{x+3}{4}\right) - 2.$$

Therefore $f^{-1}(x) = \ln\left(\dfrac{x+3}{4}\right) - 2$.

b) We have to solve for y in the equation $g(y) = x$, or $2 + \log_4(2y - 3) = x$:

$$2 + \log_4(2y - 3) = x \Rightarrow \log_4(2y - 3) = x - 2 \Rightarrow 2y - 3 = 4^{x-2} \Rightarrow 2y = 3 + 4^{x-2} \Rightarrow y = \frac{3 + 4^{x-2}}{2}$$

Therefore $g^{-1}(x) = \dfrac{3 + 4^{x-2}}{2}$.

---

7. Do exercises 1–4, 10–12, from section 4.3 (Right Triangle Trigonometry) of the text.

Solution: Please see the answer to the odd numbered exercises in the back of the textbook.