MATH 30 - Precalculus. Homework 6. Due Tu . 04/09/2019. Professor Luis Fernández

SOLUTION

1. Solve the following inequalities. Express the final answer in interval notation.

a) $(x+7)(x-3) > 0$

b) $-x^2 + x > 0$

c) $x^2 - 5x \geq -6$

d) $x^3 + 2x^2 - 4x \leq 8$

Solutions:

a) $(x+7)(x-3) > 0$. The LHS is already factored. Let us do a table of signs:

		-7	3
$(x+7)$	$-$	$+$	$+$
$(x-3)$	$-$	$-$	$+$
$(x+7)(x-3)$	$+$	$-$	$+$

Therefore $(x+7)(x-3)$ is positive in $(-\infty, -7)$ and in $(3, \infty)$, so the solution of $(x+7)(x-3) > 0$ is $(-\infty, -7) \cup (3, \infty)$

b) $-x^2 + x > 0$. Factor: $x(-x+1) > 0$. Do a table of signs:

		0	1
x	$-$	$+$	$+$
$(-x+1)$	$+$	$+$	$-$
$x(-x+1)$	$-$	$+$	$-$

Therefore $-x^2 + x > 0$ is positive in $(0, 1)$, so the solution of $-x^2 + x > 0$ is $(0, 1)$.

c) $x^2 - 5x \geq -6$. First move everything to the RHS: $x^2 - 5x + 6 \geq 0$. Then factor: $(x-2)(x-3) \geq 0$. Do a table of signs:

		2	3
$(x-2)$	$-$	$+$	$+$
$(x-3)$	$-$	$-$	$+$
$(x-2)(x-3)$	$+$	$-$	$+$

Therefore $x^2 - 5x + 6$ is positive in $(-\infty, 2)$ and $(3, \infty)$. so the solution of $x^2 - 5x + 6 \geq 0$ (which is the same as the solution of $x^2 - 5x \geq -6$) is $(-\infty, 2] \cup [3, \infty)$. Note that the brackets at 2 and 3 are square since both 2 and 3 are solutions.

d) $x^3 + 2x^2 - 4x \leq 8$. First move everything to the RHS: $x^3 + 2x^2 - 4x - 8 \leq 0$. Then factor (use synthetic division): $(x-2)(x+2)^2 \leq 0$. Do a table of signs:

		-2	2
$(x+2)^2$	$+$	$+$	$+$
$(x-2)$	$-$	$-$	$+$
$(x-2)(x+2)^2$	$-$	$-$	$+$

Therefore $x^3 + 2x^2 - 4x - 8$ is negative in $(-\infty, 2)$. so the solution is $(-\infty, 2]$. Note that the bracket at 2 is square since 2 is a solution.

2. Solve the following inequalities. Express the final answer in interval notation.

a) $\dfrac{x-2}{x+3} > 0$

b) $\dfrac{3x+5}{6-2x} \geq 0$

c) $\dfrac{x^2-3x+2}{x^2-2x-3} < 0$

d) $\dfrac{x}{x+2} \leq 2$

Solutions:

a) $\frac{x-2}{x+3} > 0$. Both the numerator and denominator are factored. Do a table of signs:

		-3		2	
$(x+3)$	$-$		$+$		$+$
$(x-2)$	$-$		$-$		$+$
$\frac{x-2}{x+3}$	$+$		$-$		$+$

Therefore the solution is $(-\infty, -3) \cup (2, \infty)$.

b) $\dfrac{3x+5}{6-2x} \geq 0$. Both the numerator and denominator are factored. The numerator is 0 at $x = -5/3$, and the denominator is 0 at $x = 3$. Do a table of signs:

		$-5/3$		3	
$(3x+5)$	$-$		$+$		$+$
$(6-2x)$	$+$		$+$		$-$
$\frac{3x+5}{6-2x}$	$-$		$+$		$-$

Therefore the solution is $[-\frac{5}{3}, 3)$. Note that $-\frac{5}{3}$ is a solution, but 3 is not.

c) $\frac{x^2-3x+2}{x^2-2x-3} < 0$. First factor both the denominator and the numerator to get $\dfrac{(x-2)(x-1)}{(x-3)(x+1)} < 0$. Do a table of signs:

		-1		1		2		3	
$(x+1)$	$-$		$+$		$+$		$+$		$+$
$(x-1)$	$-$		$-$		$+$		$+$		$+$
$(x-2)$	$-$		$-$		$-$		$+$		$+$
$(x-3)$	$-$		$-$		$-$		$-$		$+$
$\frac{(x-2)(x-1)}{(x-3)(x+1)}$	$+$		$-$		$+$		$-$		$+$

Therefore the solution is $(-1, 1) \cup (2, 3)$.

d) $\frac{x}{x+2} \leq 2$. First move everything to the RHS: $\frac{x}{x+2} - 2 \leq 0$. Then write the LHS under a common denominator: $\frac{x}{x+2} - 2 = \frac{x}{x+2} - \frac{2(x+2)}{x+2} = \frac{-x-4}{x+2}$. So the inequality is the same as $\frac{-x-4}{x+2} \leq 0$. The numerator and denominator are already factored. Do a table of signs:

		-4		-2	
$(-x-4)$	$+$		$-$		$-$
$(x+2)$	$-$		$-$		$+$
$\frac{-x-4}{x+2}$	$-$		$+$		$-$

Therefore the solution of $\frac{x}{x+2} \leq 2$ is $(-\infty, -2)$ (note that -2 is not a solution because at $x = -2$ the denominator is 0).

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3. Do exercises 11 and 51 from exercise set 2.7 in the textbook.

Solution: Please find the solution at the back of the textbook.

4. Find the domain of the following functions. Remember that for a square root to be a real number, the radicand must be greater than or equal to 0.

a) $f(x) = \sqrt{2x^2 - 5x + 2}$

b) $g(x) = \sqrt{\dfrac{x}{2x-1} - 1}$

Solutions:

a) For x to be in the domain of f, the radicand $2x^2 - 5x + 2$ must be ≥ 0. So we have to solve the inequality $2x^2 - 5x + 2 \geq 0$. First factor (use, for example, synthetic division): $2x^2 - 5x + 2 = (x-2)(2x-1)$. Then do a table of signs:

		$\frac{1}{2}$	2
$(2x-1)$	$-$	$+$	$+$
$(x-2)$	$-$	$-$	$+$
$(x-2)(2x-1)$	$+$	$-$	$+$

Therefore the domain of f is $(-\infty, \frac{1}{2}] \cup [2, \infty)$.

b) For x to be in the domain of g, the radicand $\dfrac{x}{2x-1} - 1$ must be ≥ 0. So we have to solve the inequality $\dfrac{x}{2x-1} - 1 \geq 0$. First write the LHS under the same denominator:

$$\frac{x}{2x-1} - 1 = \frac{x}{2x-1} - \frac{2x-1}{2x-1} = \frac{-x+1}{2x-1},$$

so we need to solve $\dfrac{-x+1}{2x-1} \geq 0$. Both numerator and denominator are factored, so do a table of signs:

		$\frac{1}{2}$	1
$(2x-1)$	$-$	$+$	$+$
$(-x+1$	$+$	$+$	$-$
$(x-2)(2x-1)$	$-$	$+$	$-$

Therefore the domain of g is $[\frac{1}{2}, 1]$.

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5. Solve the following inequalities. Express the final answer in interval notation.

a) $x^3 + 2x^2 - 4x - 8 > 0$

b) $x^4 \geq -4x^2$

c) $\dfrac{1}{x+1} < \dfrac{2}{x-1}$

d) $\dfrac{x^3 - 4x^2 + 6x - 2}{x^2 - 5x + 6} \leq x$

Solutions:

a) $x^3 + 2x^2 - 4x - 8 > 0$. Factor (use synthetic division): $x^3 + 2x^2 - 4x - 8 = (x-2)(x+2)^2$. Do a table of signs:

		-2	2
$(x+2)^2$	$+$	$+$	$+$
$(x-2)$	$-$	$-$	$+$
$(x-2)(x+2)^2$	$-$	$-$	$+$

Therefore the solution is $(2, \infty)$.

b) $x^4 \geq -4x^2$. Bring everything to the LHS: $x^4 + 4x^2 \geq 0$. Then factor the LHS: $x^4 + 4x^2 = x^2(x^2 + 4)$. Note that $x^2 + 4$ is positive for any real value of x. Do a table of signs:

		0	
x^2	$+$		$+$
(x^2+4)	$+$		$+$
$x^2(x^2+4)$	$+$		$+$

Therefore the solution is $(-\infty, \infty)$ (in other words, every real number is a solution).

c) $\dfrac{1}{x+1} < \dfrac{2}{x-1}$. Bring everything to the LHS: $\dfrac{1}{x+1} - \dfrac{2}{x-1} < 0$. Then write the LHS with the same denominator:

$$\frac{1}{x+1} - \frac{2}{x-1} = \frac{x-1}{(x+1)(x-1)} - \frac{2(x+1)}{(x+1)(x-1)} = \frac{-x-3}{(x+1)(x-1)} < 0.$$

Both numerator and denominator are factored, so do a table of signs

		-3	-1	1
$(-x-3)$	$+$	$-$	$-$	$-$
$(x+1)$	$-$	$-$	$+$	$+$
$(x-1)$	$-$	$-$	$-$	$+$
$\dfrac{-x-3}{(x-1)(x+1)}$	$+$	$-$	$+$	$-$

Therefore the solution is $(-3, -1) \cup (1, \infty)$.

d) $\dfrac{x^3-4x^2+6x-2}{x^2-5x+6} \leq x$. Bring everything to the LHS: $\dfrac{x^3-4x^2+6x-2}{x^2-5x+6} - x \leq 0$. Then put the LHS under a common denominator:

$$\frac{x^3-4x^2+6x-2}{x^2-5x+6} - x = \frac{x^3-4x^2+6x-2}{x^2-5x+6} - \frac{x(x^2-5x+6)}{x^2-5x+6} = \frac{x^3-4x^2+6x-2}{x^2-5x+6} - \frac{x^3-5x^2+6x}{x^2-5x+6} = \frac{x^2-2}{x^2-5x+6} \leq 0.$$

Factor: the zeros of x^2-2 are: $x^2-2=0 \Rightarrow x^2=2 \Rightarrow x = \pm\sqrt{2}$, so $x^2-2 = (x-\sqrt{2})(x+\sqrt{2})$ (by the Factor Theorem). The denominator factors as $x^2-5x+6 = (x-3)(x-2)$, and therefore we have to solve the inequality

$$\frac{(x-\sqrt{2})(x+\sqrt{2})}{(x-3)(x-2)} \leq 0.$$

Do a table of signs:

		$-\sqrt{2}$	$\sqrt{2}$	2	3
$(x+\sqrt{2})$	$-$	$+$	$+$	$+$	$+$
$(x-\sqrt{2})$	$-$	$-$	$+$	$+$	$+$
$(x-2)$	$-$	$-$	$-$	$+$	$+$
$(x-3)$	$-$	$-$	$-$	$-$	$+$
$\dfrac{(x-\sqrt{2})(x+\sqrt{2})}{(x-3)(x-2)}$	$+$	$-$	$+$	$-$	$+$

Therefore the solution is $[-\sqrt{2}, \sqrt{2}] \cup (2, 3)$. Note that $\sqrt{2}$ and $-\sqrt{2}$ are solutions since at these values the numerator is 0 (and $0 \leq 0$), whereas 2 and 3 are not since at these values the denominator is 0, so the fraction is undefined.

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6. Do exercises 23, 25, 27, 41, 43, 67, 69, 71 from exercise set 4.4 in the textbook.

Solutions: See the answers in the back of the book.