MATH 30 - Precalculus. Homework 4. Due Tu. 03/14/2019. Professor Luis Fernández

SOLUTION

Write your answers in other sheets and **STAPLE this one to your other sheets.**

1. For each of the following functions, find

(i) The end behaviour of the graph.

(ii) The y-intercept.

(iii) For exercises **a)**, **b)**, **c)**, the x-intercepts with their multiplicity and the local behaviour at the x-intercepts.

(iv) Do a sketch of the graph, in the graph paper provided, of each function in **a)**, **b)**, **c)**.

(v) Do the graphs of all the functions using any graphing device; for example, go to:

http://www.mathsisfun.com/data/function-grapher.php

Compare **a)**, **b)**, **c)**, with your sketches. Also check that the end behaviour of the graphs that you found in part **(i)** are all correct.

a) $f(x) = 2(x-2)^2(x+1)$ **b)** $f(x) = -2x^2(x-2)(x+2)^2$ **c)** $f(x) = 3x(x+1)^2(x-1)^3$

d) $f(x) = -x^4 + 5x^2 + x$ **e)** $f(x) = 2x^4 + 3x^2 - 3$ **f)** $f(x) = -3x^3 + 3x^2 - x + 1$

Solution: Do the graphs with your favorite graphing device to see the solution.

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2. Divide using long division. State the quotient $q(x)$ and the remainder $r(x)$. Then write the solution in two different ways:

1. As $D(x) = d(x)q(x) + r(x)$.
2. As $\dfrac{D(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

[Where $D(x)$ is the dividend (the polynomial that is being divided; in other words, the numerator) and $d(x)$ is the divisor (the polynomial that divides; in other words, the denominator).]

a) $\dfrac{x^3 - 2x^2 - 5x + 6}{x+2}$ **b)** $\dfrac{3x^4 - 2x^3 - 7x^2 + x - 2}{x^2 - 2x + 3}$

c) $\dfrac{x^5 + x^4 - x^3 - x^2 + 3x - 1}{x^2 + x + 1}$ **d)** $\dfrac{x^4 - 2x^2 - 5x + 6}{x - 3}$

e) $\dfrac{3x^6 - 2x^3 - 7x^2 - 2}{x^2 - x + 2}$ **f)** $\dfrac{x^7 - 1}{x - 1}$

Solution:

a)

$$
\begin{array}{r|l}
 & x^2 - 4x + 3 \\
\hline
x+2 & x^3 - 2x^2 - 5x + 6 \\
- & x^3 + 2x^2 \\
\hline
 & \quad -4x^2 - 5x \\
- & \quad -4x^2 - 8x \\
\hline
 & \qquad 3x + 6 \\
- & \qquad 3x + 6 \\
\hline
 & \qquad\quad 0
\end{array}
$$

Therefore $q(x) = x^2 - 4x + 3$ and $r(x) = 0$

1. $x^3 - 2x^2 - 5x + 6 = (x+2)(x^2 - 4x + 3) + 0$
2. $\dfrac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3} = (x+2)$

b)

$$
\begin{array}{r|l}
 & 3x^2 + 4x - 8 \\
\hline
x^2 - 2x + 3 & 3x^4 - 2x^3 - 7x^2 + x - 2 \\
- & 3x^4 - 6x^3 + 9x^2 \\
\hline
 & \quad 4x^3 - 16x^2 + x \\
- & \quad 4x^3 - 8x^2 + 12x \\
\hline
 & \qquad -8x^2 - 11x - 2 \\
- & \qquad -8x^2 + 16x - 24 \\
\hline
 & \qquad\quad -27x + 22
\end{array}
$$

Therefore $q(x) = 3x^2 + 4x - 8$ and $r(x) = -27x + 22$

1. $3x^4 - 2x^3 - 7x^2 + x - 2$
   $= (x^2 - 2x + 3)(3x^2 + 4x - 8) + (-27x + 22)$
2. $\dfrac{3x^4 - 2x^3 - 7x^2 + x - 2}{x^2 - 2x + 3}$
   $= (3x^2 + 4x - 8) + \dfrac{-27x + 22}{x^2 - 2x + 3}$

c) (Division skipped. But you can check your answer, for example, at http://webgraphing.com/polydivision.jsp).

Answer: $q(x) = x^3 - 2x + 1$ and $r(x) = 4x - 2$, so

1. $x^5 + x^4 - x^3 - x^2 + 3x - 1 = (x^2 + x + 1)(x^3 - 2x + 1) + (4x - 2)$
2. $\dfrac{x^5 + x^4 - x^3 - x^2 + 3x - 1}{x^2 + x + 1} = (3x^3 - 2x^2 - 2x + 3) + \dfrac{2x - 4}{x^2 + x + 1}$

d) (Division skipped. But you can check your answer, for example, at http://webgraphing.com/polydivision.jsp).

Answer: $q(x) = x^3 + 3x^2 + 7x + 16$ and $r(x) = 54$, so

1. $x^4 - 2x^2 - 5x + 6 = (x - 3)(x^3 + 3x^2 + 7x + 16) + 54$
2. $\dfrac{x^4 - 2x^2 - 5x + 6}{x - 3} = (x^3 + 3x^2 + 7x + 16) + \dfrac{54}{x - 3}$

e) (Division skipped. But you can check your answer, for example, at http://webgraphing.com/polydivision.jsp).

Answer: $q(x) = 3x^4 + 3x^3 - 3x^2 - 11x - 12$ and $r(x) = 10x + 22$, so

1. $3x^6 - 2x^3 - 7x^2 - 2 = (x^2 - x + 2)(3x^4 + 3x^3 - 3x^2 - 11x - 12) + (10x + 22)$
2. $\dfrac{3x^6 - 2x^3 - 7x^2 - 2}{x^2 - x + 2} = (3x^4 + 3x^3 - 3x^2 - 11x - 12) + \dfrac{10x + 22}{x^2 - x + 2}$

f)

$$
\begin{array}{rrcrcrcrcrcrcrcr}
 & x^7 & + & x^6 & + & x^5 & + & x^4 & + & x^3 & + & x^2 & + & x & + & 1 \\ \cline{2-16}
x-1\ \big) & x^7 & + & 0\cdot x^6 & + & 0\cdot x^5 & + & 0\cdot x^4 & + & 0\cdot x^3 & + & 0\cdot x^2 & + & 0\cdot x^1 & - & 1 \\
- & x^7 & - & x^6 \\ \cline{2-16}
 & & - & x^6 & + & 0\cdot x^5 \\
 & & & x^6 & - & x^5 \\ \cline{4-16}
 & & & & - & x^5 & + & 0\cdot x^4 \\
 & & & & & x^5 & - & x^4 \\ \cline{6-16}
 & & & & & & - & x^4 & + & 0\cdot x^3 \\
 & & & & & & & x^4 & - & x^3 \\ \cline{8-16}
 & & & & & & & & - & x^3 & + & 0\cdot x^2 & + \\
 & & & & & & & & & x^3 & - & x^2 \\ \cline{10-16}
 & & & & & & & & & & - & x^2 & + & 0\cdot x \\
 & & & & & & & & & & & x^2 & - & x \\ \cline{12-16}
 & & & & & & & & & & & & - & x & - & 1 \\
 & & & & & & & & & & & & & x & - & 1 \\ \cline{14-16}
 & & & & & & & & & & & & & & & 0
\end{array}
$$

Answer: $q(x) = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ and $r(x) = 0$, so

1. $x^7 - 1 = (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$
2. $\dfrac{x^7 - 1}{x - 1} = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

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3. Divide using synthetic division. State the quotient $q(x)$ and the remainder $r(x)$. Then write the solution in two different ways:

1. As $D(x) = d(x)q(x) + r(x)$.
2. As $\dfrac{D(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

[Where $D(x)$ is the dividend (the polynomial that is being divided; in other words, the numerator) and $d(x)$ is the divisor (the polynomial that divides; in other words, the denominator).]

a) $\dfrac{x^3 - 2x^2 - 5x + 6}{x - 3}$

b) $\dfrac{-2x^3 - 7x^2 + x - 2}{x + 1}$

c) $\dfrac{x^4 - x^3 + x - 1}{x - 2}$

d) $\dfrac{x^7 - 1}{x - 1}$

Solution:

a)

$$\begin{array}{r|rrrr} & 1 & -2 & 5 & 6 \\ 3 & & 3 & 3 & 24 \\ \hline & 1 & 1 & 8 & 30 \end{array}$$

So $q(x) = x^2 + x + 8$ and $r(x) = 30$.

1. $x^3 - 2x^2 - 5x + 6 = (x-3)(x^2 + x + 8) + 30$
2. $\dfrac{x^3 - 2x^2 - 5x + 6}{x-3} = x^2 + x + 8 + \dfrac{30}{x-3}$

b)

$$\begin{array}{r|rrrr} & -2 & -7 & 1 & -2 \\ -1 & & 2 & 5 & -6 \\ \hline & -2 & -5 & 6 & -8 \end{array}$$

So $q(x) = -2x^2 - 5x + 6$ and $r(x) = -8$.

1. $-2x^3 - 7x^2 + x - 2 = (x+1)(-2x^2 - 5x + 6) - 8$
2. $\dfrac{-2x^3 - 7x^2 + x - 2}{x+1} = -2x^2 - 5x + 6 + \dfrac{-8}{x+1}$

c)

$$\begin{array}{r|rrrrr} & 1 & -1 & 0 & 1 & -1 \\ 2 & & 2 & 2 & 4 & 10 \\ \hline & 1 & 1 & 2 & 5 & 9 \end{array}$$

So $q(x) = x^3 + x^2 + 2x + 5$ and $r(x) = 9$.

1. $x^4 - x^3 + x - 1 = (x-2)(x^3 + x^2 + 2x + 5) + 9$
2. $\dfrac{x^4 - x^3 + x - 1}{x-2} = x^3 + x^2 + 2x + 5 + \dfrac{9}{x-2}$

d)

$$\begin{array}{r|rrrrrrrr} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

So $q(x) = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ and $r(x) = 0$.

1. $x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$
2. $\dfrac{x^7 - 1}{x-1} = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

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4. Use synthetic division and the remainder theorem to find the indicated function value.

a) $f(x) = x^3 - 4x^2 + x + 2$; find $f(3)$.

b) $f(x) = -2x^4 - x^2 + x - 2$; find $f(-1)$.

c) $f(x) = x^5 - 4x^2 + 1$; find $f(2)$.

d) $f(x) = -x^4 - 5x^3 - x^2 + 3x + 2$; find $f\left(\frac{1}{2}\right)$.

Solution:

Synthetic divisions skipped. For example, you can use http://www.mathcelebrity.com/syndiv.php to check it.

a) Answer: $f(3) = -4$.

b) Answer: $f(-1) = -6$.

c) Answer: $f(2) = 17$.

d) Answer: $f(1/2) = \frac{41}{16}$.

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5. Solve the equation $2x^3 - 3x^2 - 11x + 6 = 0$ given that -2 is a zero of $f(x) = 2x^3 - 3x^2 - 11x + 6$.

Solution:

Since -2 is a zero of $f(x)$, we know that we can divide $f(x)$ by $x+2$. Do it using synthetic division:

$$\begin{array}{r|rrrr} & 2 & -3 & -11 & 6 \\ -2 & & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

Therefore $2x^3 - 3x^2 - 11x + 6 = (x+2)(2x^2 - 7x + 3)$. So solving $2x^3 - 3x^2 - 11x + 6 = 0$ is the same as solving $(x+2)(2x^2 - 7x + 3) = 0$. We have two possibilities: $x + 2 = 0$ (which gives the solution $x = -2$ we already know) and $2x^2 - 7x + 3 = 0$. The solutions of this equation can be found, for example, using the quadratic formula:

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4},$$

which gives $x = 3$ and $x = \frac{1}{2}$.

Therefore the solutions of the equation $2x^3 - 3x^2 - 11x + 6 = 0$ are -2, 3 and $\frac{1}{2}$.

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6. Solve the equation $3x^3 + 7x^2 - 22x - 8 = 0$ given that $-\frac{1}{3}$ is a root.

Solution:

Since $-\frac{1}{3}$ is a root (which is the same as a zero of the polynomial $f(x)$), we know that we can divide $f(x)$ by $x + \frac{1}{3}$. Do it using synthetic division:

$$\begin{array}{r|rrrr} & 3 & 7 & -22 & -8 \\ -1/3 & & -1 & -2 & 8 \\ \hline & 3 & 6 & -24 & 0 \end{array}$$

Therefore $3x^3 + 7x^2 - 22x - 8 = (x + \frac{1}{3})(3x^2 + 6x - 24) = 3(x + \frac{1}{3})(x^2 + 2x - 8) = 3(x + \frac{1}{3})(x-2)(x+4)$, and therefore the solutions of the equation $3x^3 + 7x^2 - 22x - 8 = 0$ are $-\frac{1}{3}$, 2 and -4.

7. The remainder from dividing a polynomial $p(x)$ by $(x + 4)$ is 3. How much is $p(-4)$? Which theorem are you using?

Solution:

The **remainder theorem** states that if a polynomial f is divided by $x - c$ then the remainder is $f(c)$. In this case, we know that when we divide $p(x)$ by $(x + 4)$ (which is the same as $x - (-4)$) the remainder is 3. Therefore, $p(-4) = 3$.

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8. The remainder from dividing a polynomial $p(x)$ by $\left(x - \frac{1}{2}\right)$ is $\frac{11}{17}$. How much is $p\left(\frac{1}{2}\right)$? Which theorem are you using?

Solution:

The **remainder theorem** states that if a polynomial f is divided by $x - c$ then the remainder is $f(c)$. In this case, we know that when we divide $p(x)$ by $(x - \frac{1}{2})$ the remainder is $\frac{11}{17}$. Therefore, $p(\frac{1}{2}) = \frac{11}{17}$.

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9. What is the remainder you would get it you divide the polynomial $f(x) = x^{103} + x^{50} + 2$ by $(x - 1)$?

NOTE: you do not really have to divide this huge polynomial; you can do this in your head if you use the appropriate theorem!

Solution:

The **remainder theorem** states that if a polynomial f is divided by $x - c$ then the remainder is $f(c)$. In this case, if we divide $f(x) = x^{103} + x^{50} + 2$ by $x - 1$ (in other words, $c = 1$) then the remainder will be $f(1)$. But $f(1)$ is very easy to find: $f(1) = 1^{103} + 1^{50} + 2 = 1 + 1 + 2 = 4$. Therefore, the remainder is 4.