## NAME:\_

Write your answers in other sheets and STAPLE this one to your other sheets.

- 1. For each of the following functions, find
- (i) The end behaviour of the graph.
- (ii) The *y*-intercept.
- (iii) For exercises a), b), c), the x-intercepts with their multiplicity and the local behaviour at the x-intercepts.
- (iv) Do a sketch of the graph, in the graph paper provided, of each function in a), b), c).
- (v) Do the graphs of all the functions using any graphing device; for example, go to:

http://www.mathsisfun.com/data/function-grapher.php

Compare **a**), **b**), **c**), with your sketches. Also check that the end behaviour of the graphs that you found in part (i) are all correct.

- a)  $f(x) = 2(x-2)^2(x+1)$  b)  $f(x) = -2x^2(x-2)(x+2)^2$  c)  $f(x) = 3x(x+1)^2(x-1)^3$ d)  $f(x) = -x^4 + 5x^2 + x$  e)  $f(x) = 2x^4 + 3x^2 - 3$  f)  $f(x) = -3x^3 + 3x^2 - x + 1$
- **2.** Divide using long division. State the quotient q(x) and the remainder r(x). Then write the solution in two different ways:
  - 1. As D(x) = d(x)q(x) + r(x). 2. As  $\frac{D(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$ .

[Where D(x) is the dividend (the polynomial that is being divided; in other words, the numerator) and d(x) is the divisor (the polynomial that divides; in other words, the denominator).]

- a)  $\frac{x^{3} 2x^{2} 5x + 6}{x + 2}$ b)  $\frac{3x^{4} - 2x^{3} - 7x^{2} + x - 2}{x^{2} - 2x + 3}$ c)  $\frac{x^{5} + x^{4} - x^{3} - x^{2} + 3x - 1}{x^{2} + x + 1}$ d)  $\frac{x^{4} - 2x^{2} - 5x + 6}{x - 3}$ e)  $\frac{3x^{6} - 2x^{3} - 7x^{2} - 2}{x^{2} - x + 2}$ f)  $\frac{x^{7} - 1}{x - 1}$
- **3.** Divide using synthetic division. State the quotient q(x) and the remainder r(x). Then write the solution in two different ways:
  - 1. As D(x) = d(x)q(x) + r(x). 2. As  $\frac{D(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$ .

[Where D(x) is the dividend (the polynomial that is being divided; in other words, the numerator) and d(x) is the divisor (the polynomial that divides; in other words, the denominator).]

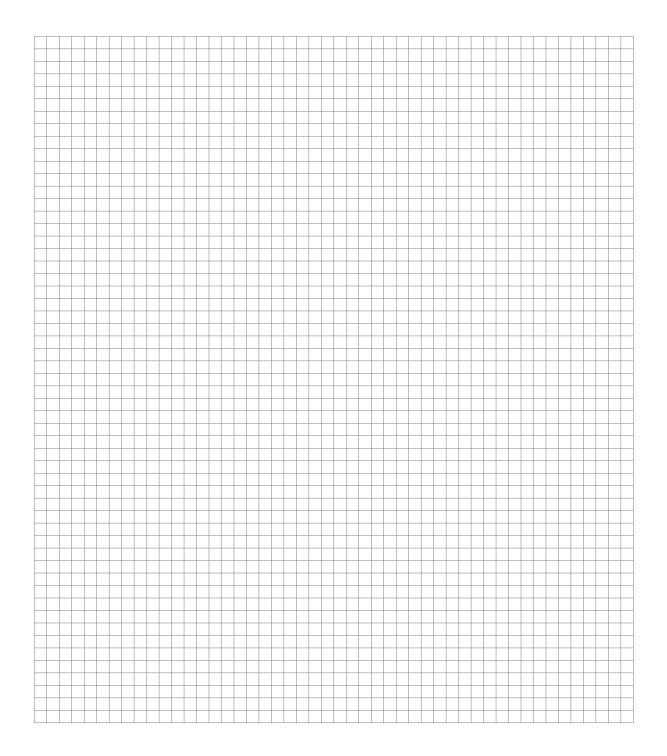
a) $\frac{x^3 - 2x^2 - 5x + 6}{x - 3}$	b) $\frac{-2x^3 - 7x^2 + x - 2}{x + 1}$
c) $\frac{x^4 - x^3 + x - 1}{x - 2}$	d) $\frac{x^7 - 1}{x - 1}$

4. Use synthetic division and the remainder theorem to find the indicated function value.

a)  $f(x) = x^3 - 4x^2 + x + 2$ ; find f(3). b)  $f(x) = -2x^4 - x^2 + x - 2$ ; find f(-1). c)  $f(x) = x^5 - 4x^2 + 1$ ; find f(2). d)  $f(x) = -x^4 - 5x^3 - x^2 + 3x + 2$ ; find  $f\left(\frac{1}{2}\right)$ .

5. Solve the equation  $2x^3 - 3x^2 - 11x + 6 = 0$  given that -2 is a zero of  $f(x) = 2x^3 - 3x^2 - 11x + 6$ .

- 6. Solve the equation  $3x^3 + 7x^2 22x 8 = 0$  given that  $-\frac{1}{3}$  is a root.
- 7. The remainder from dividing a polynomial p(x) by (x+4) is 3. How much is p(-4)? Which theorem are you using?
- 8. The remainder from dividing a polynomial p(x) by  $\left(x-\frac{1}{2}\right)$  is  $\frac{11}{17}$ . How much is  $p\left(\frac{1}{2}\right)$ ? Which theorem are you using?
- 9. What is the remainder you would get it you divide the polynomial  $f(x) = x^{103} + x^{50} + 2$  by (x 1)? NOTE: you do not really have to divide this huge polynomial; you can do this in your head if you use the appropriate theorem!



		_	_		+-		-			_									_			_			-	-	+		_	+
			-							_									_						-	-	+			+
																									-	-	-			
			-																						-		+			+
			-																						-	-	+			+
																											-			
							_			_										_										
					_		_			_										_					_	_	_			
			_		_					_															_	_	_		_	
		_	_		_		_		_	_		_								_		_			_	_	_		_	
	+	+	_	+	-	$\left  \cdot \right $	_	$\vdash$	++	_	$\vdash$				$\left  \cdot \right $		_	+	_		$\vdash$	_		$\vdash$	_	_	+	$\vdash$	_	+
	+		+		+-	$\left  \right $	-	$\vdash$	+	_				_	$\left  \cdot \right $	+	_	+	_	-				$\vdash$		-	-	$\vdash$	_	+
	+	+	_	+	+-	$\vdash$	+	$\vdash$	++		+				+	+	_	+		-	$\vdash$			$\vdash$	-	+	+-	$\vdash$		+
	+	+		+	+-	$\left  \cdot \right $	+	$\vdash$	++		+				$\vdash$	+	_	+		-	$\left  \right $		$\left  \right $	$\vdash$		+	+	$\vdash$		+
	+	+	-	+	+-	$\left  \cdot \right $	+	$\vdash$	++		++				$\vdash$	+		+		-	$\vdash$			$\vdash$		-	+	$\vdash$		+
	++	+	+	+	+-	$\vdash$	+	$\vdash$	++		++		_		+	+		+		+	$\vdash$	-	$\square$	$\vdash$	+	+	+	$\vdash$		+
	++	+	+	++	+-	$\vdash$	+	$\vdash$	++		++				$\vdash$	+		+		+	$\vdash$			$\vdash$	-	+	+	$\vdash$		+
			+		+	$\vdash$	+	$\vdash$	++	-							-	+		-	$\vdash$	-		$\vdash$	-	+	+	$\vdash$	-	+
	++		+		+	$\vdash$	+	$\vdash$			++					+	+		-	+		-		$\vdash$	+	+	+	$\vdash$	+	+
	++	+	+	++	+		+		++		+					++		+		-				$\vdash$	-	+	+	$\vdash$	-	+
																											-			
					-																				-	-	+			
			-																						-	-	-			$\square$
																											-			
																				_						_				

		_	_		+-		-			_									_			_			-	-	+		_	+
			-							_									_						-	-	+			+
																									-	-	-			
			-																						-		+			+
			-				-																		-	-	+			+
																											1			
							_			_										_										
					_		_			_										_					_	_	_			
			_		_					_															_	_	_		_	
		_	_		_		_		_	_		_								_		_			_	_	_		_	
	+	+	_	+	-	$\vdash$	-	$\vdash$	++	_	$\vdash$				$\left  \cdot \right $		_	+	_		$\square$	_		$\vdash$	_	_	+	$\vdash$	_	+
	+		+		+-	$\left  \right $	-	$\vdash$	+	_				_	$\left  \cdot \right $	+	_	+	_	-				$\vdash$		-	-	$\vdash$	_	+
	+	+	_	+	+-	$\vdash$	+	$\vdash$	++		+				+	+	_	+	_	-	$\vdash$			$\vdash$	-	+	+-	$\vdash$		+
	+	+		+	+-	$\left  \cdot \right $	+	$\vdash$	++		+				$\vdash$	+		+		-			$\left  \right $	$\vdash$		+	+	$\vdash$		+
	+	+	-	+	+-	$\left  \cdot \right $	+	$\vdash$	++		++				$\vdash$	+		+		-	$\vdash$			$\vdash$		-	+	$\vdash$		+
	++	+	+	+	+-	$\vdash$	+	$\vdash$	++		++		_		+	+		+		+	$\vdash$	-	$\square$	$\vdash$	+	+	+	$\vdash$		+
	++	+	+	++	+-	$\vdash$	+	$\vdash$	++		++				$\vdash$	+		+		+	$\vdash$			$\vdash$	-	+	+	$\vdash$		+
			+		+	$\vdash$	+	$\vdash$	++	-								+		-	$\vdash$	-		$\vdash$	-	+	+	$\vdash$	-	+
	++		+		+	$\vdash$	+	$\vdash$			++					+	+		-	+		-		$\vdash$	+	+	+	$\vdash$	+	+
	++	+	+	++	+		+		++		+				+	+		+		-				$\vdash$	-	+	+	$\vdash$	-	+
																											-			
					-																				-	-	+			
			-																						-		+			$\square$
																											-			
																				_						_				