MATH 30 - Precalculus. Homework 3. Due Th. 02/21/2019. Professor Luis Fernández

SOLUTION

1. Recall that to show that a function g is the inverse of a function f one needs to show that $f(g(x)) = x$ and that $g(f(x)) = x$. To do this,

1. Find $f(g(x))$ and simplify and see that you get x.
2. Find $g(f(x))$ and simplify and see that you get x.

For the following, show that g is the inverse of f.

a) $f(x) = 4x - 7$ and $g(x) = \dfrac{x+7}{4}$.

b) $f(x) = \dfrac{2}{x-5}$ and $g(x) = \dfrac{2}{x} + 5$.

c) $f(x) = -3x + 1$ and $g(x) = \dfrac{x-1}{-3}$.

d) $f(x) = \dfrac{x-2}{2x+1}$ and $g(x) = \dfrac{-x-2}{2x-1} + 5$.

Solution:

a)
1. $f(g(x)) = f(\frac{x+7}{4}) = 4 \cdot \frac{x+7}{4} - 7 = (x+7) - 7 = x$. YES.
2. $g(f(x)) = g(4x-7) = \dfrac{(4x-7)+7}{4} = \frac{4x}{4} = x$. YES.

b)
1. $f(g(x)) = f(\frac{2}{x} + 5) = \dfrac{2}{\left(\dfrac{2}{x} + 5\right) - 5} = \dfrac{2}{\dfrac{2}{x}} = x$. YES.
2. $g(f(x)) = g(\frac{2}{x-5}) = \dfrac{2}{\dfrac{2}{x-5}} + 5 = \dfrac{2(x-5)}{2} + 5 = (x-5) + 5 = x$. YES.

c) and **d)**: proceed in the same way.

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2. Find the inverse of the following functions.

a) $f(x) = 2x - 1$

b) $g(x) = \dfrac{1}{x} + 1$

c) $h(x) = x^2 - 4$, with domain $(-\infty, 0]$ (so $x \leq 0$)

d) $i(x) = \dfrac{x-1}{x+1}$.

Solution:

a) We need to solve $f(y) = x$ for y, i.e. solve $2y - 1 = x$ for y.

$$2y - 1 = x \quad \Rightarrow \quad 2y = x + 1 \quad \Rightarrow \quad y = \frac{x+1}{2}.$$

Therefore the inverse of f is $f^{-1}(x) = \frac{x+1}{2}$ (or, if you prefer, $f^{-1}(y) = \frac{y+1}{2}$).

b) We need to solve $g(y) = x$ for y, i.e. solve $\frac{1}{y} + 1 = x$ for y.

$$\frac{1}{y} + 1 = x \quad \Rightarrow \quad \frac{1}{y} = x - 1 \quad \Rightarrow \quad y = \frac{1}{x-1}.$$

Therefore the inverse of g is $g^{-1}(x) = \frac{1}{x-1}$.

c) We need to solve $h(y) = x$ for y, i.e. solve $y^2 - 4 = x$ for y, where we know that $y \leq 0$.

$$y^2 - 4 = x \quad \Rightarrow \quad y^2 = x + 4 \quad \Rightarrow \quad y = \pm\sqrt{x+4},$$

but since we know that y is negative, the only possible solution is $y = -\sqrt{x+4}$. Therefore the inverse of h is $h^{-1}(x) = -\sqrt{x+4}$.

d) We need to solve $i(y) = x$ for y, i.e. solve $\frac{y-1}{y+1} = x$ for y.

$$\frac{y-1}{y+1} = x \quad\Rightarrow\quad y-1 = x(y+1) \quad\Rightarrow\quad y-1 = xy+x \quad\Rightarrow\quad y-xy = x+1 \quad\Rightarrow\quad y(1-x) = x+1 \quad\Rightarrow\quad y = \frac{x+1}{1-x}.$$

Therefore the inverse of i is $i^{-1}(x) = \frac{x+1}{1-x}$.

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3. Recall that the range of a function is the domain of its inverse. Using this fact, find the range of the functions f, g and h of the previous exercise.

a) The domain of f^{-1} is $\mathbb{R}$, so the range of f is $\mathbb{R}$.

b) The domain of g^{-1} is $(-\infty, 1) \cup (1, \infty)$. Therefore the range of g is $(-\infty, 1) \cup (1, \infty)$.

c) The domain of h^{-1} is $[-4, \infty)$. Therefore the range of h is $[-4, \infty)$.

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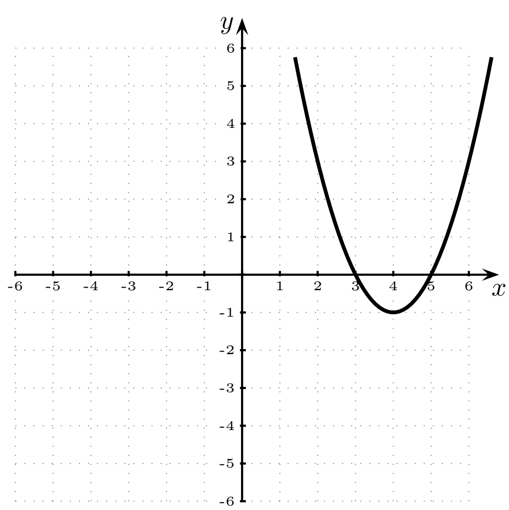
4. For the following quadratic functions,

- Find the vertex and x- and y-intercepts.
- Draw the graph in the graph paper provided (or on your own graph paper).
- Give the equation of the axes of symmetry.
- Determine the function's domain and range.

a) $f(x) = (x-4)^2 - 1$.



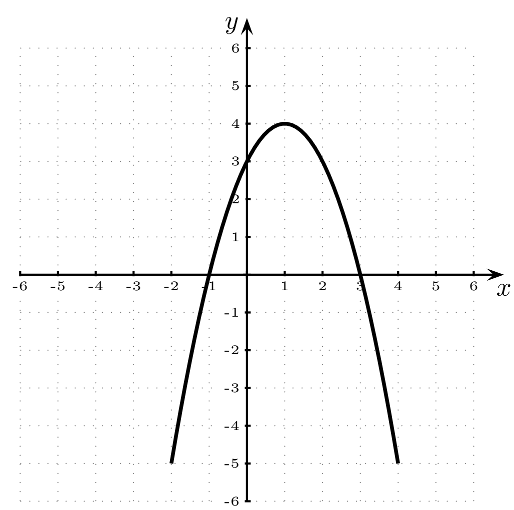
Vertex: $(4, -1)$

y-intercept: 15. **x-intercepts:** $3, 5$

Axes of symmetry: $x = 4$

Domain: $\mathbb{R}$. **Range:** $[-1, \infty)$

b) $g(x) = 4 - (x-1)^2$.



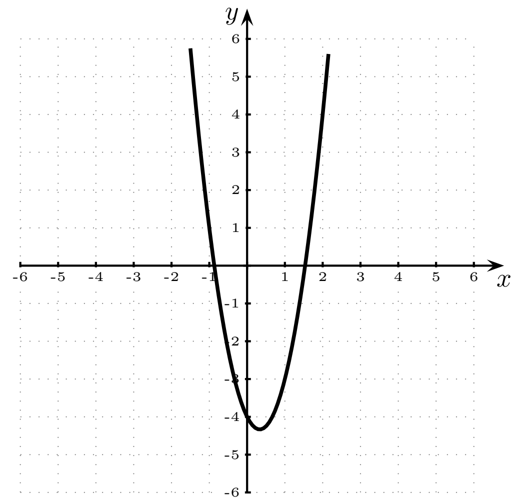
Vertex: $(1, 4)$

y-intercept: 3. **x-intercepts:** $-1, 3$

Axes of symmetry: $x = 1$

Domain: $\mathbb{R}$. **Range:** $(-\infty, 4]$

c) $h(x) = 3x^2 - 2x - 4$.



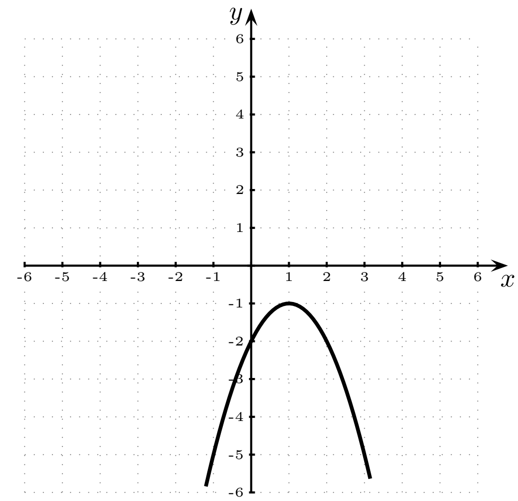
Vertex: $(1/3, -13/3)$

y**-intercept:** -4. x**-intercepts:** $\dfrac{1 \pm \sqrt{13}}{3}$

Axes of symmetry: $x = 1/3$

Domain: $\mathbb{R}$. **Range:** $[-13/3, \infty)$

d) $i(x) = 2x - x^2 - 2$.



Vertex: $(1, -1)$

y**-intercept:** -2. x**-intercepts:** NONE

Axes of symmetry: $x = 1$

Domain: $\mathbb{R}$. **Range:** $(-\infty, 1]$

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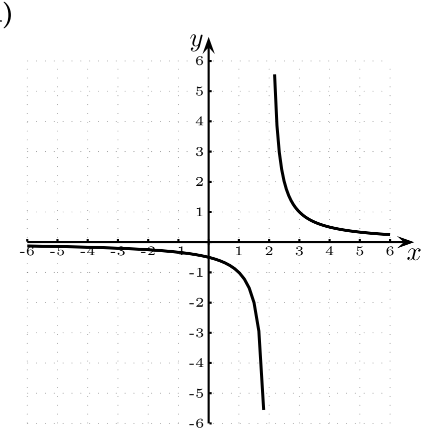
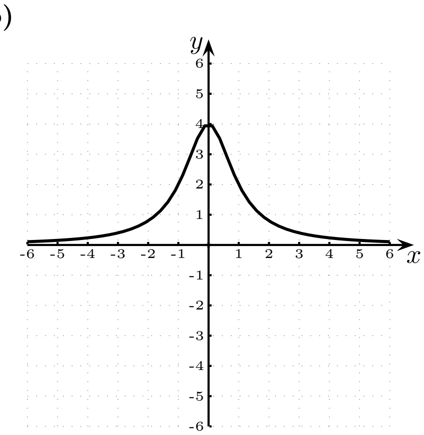
5. [BONUS] Use a graphing device to find functions whose graphs look roughly like the following. (HOW? Play!)

a)



Solution: For example, $f(x) = \dfrac{1}{x-2}$

b)



Solution: For example, $f(x) = \dfrac{1}{x^2+1}$

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6. Find an angle between 0 and 2π that is coterminal with the following angles:

a) $\dfrac{27\pi}{4}$ **b)** $\dfrac{7\pi}{4} + 5\pi$ **c)** $\dfrac{-13\pi}{3}$

a) $\dfrac{27\pi}{4} = \dfrac{27}{4}\pi = \left(6 + \dfrac{3}{4}\right)\pi = 6\pi + \dfrac{3\pi}{4}$, which is coterminal with $\dfrac{3\pi}{4}$. **b)** $\dfrac{7\pi}{4} + 5\pi = \dfrac{7\pi}{4} + \pi + 4\pi = \dfrac{11\pi}{4} + 4\pi$, which is coterminal with $\dfrac{11\pi}{4}$. **c)** $\dfrac{-13\pi}{3} = \left(-4 - \dfrac{1}{3}\right)\pi = -4\pi - \dfrac{\pi}{3}$, which is coterminal with $\dfrac{-\pi}{3}$, which is coterminal with $\dfrac{-\pi}{3} + 2\pi = \dfrac{5\pi}{3}$.