MATH 30 - Precalculus. Homework 2. Due Th. 02/14/2019. Professor Luis Fernández

NAME:____

Write your answers in other sheets and **STAPLE this one to your other sheets.**

1. Let $f(x) = x^2 + x - 2$ and $g(x) = \dfrac{x}{x^2+3}$. Find, **without simplifying**,

a) $f(x+4) = (x+4)^2 + (x+4) - 2$

b) $f(x^2-1) = (x^2-1)^2 + (x^2-1) - 2$

c) $g(x^2) = \dfrac{x^2}{(x^2)^2+3}$

d) $g(tail) = \dfrac{tail}{(tail)^2+3}$

e) $f(g(x)) = (g(x))^2 + (g(x)) - 2 = \left(\dfrac{x}{x^2+3}\right)^2 + \left(\dfrac{x}{x^2+3}\right) - 2$

f) $g(f(x)) = \dfrac{f(x)}{(f(x))^2+3} = \dfrac{x^2+x-2}{(x^2+x-2)^2+3}$

g) $f(f(g(x)) = \left[\left(\dfrac{x}{x^2+3}\right)^2 + \left(\dfrac{x}{x^2+3}\right) - 2\right]^2 + \left[\left(\dfrac{x}{x^2+3}\right)^2 + \left(\dfrac{x}{x^2+3}\right) - 2\right] - 2$

h) $\dfrac{\dfrac{x^2+x-2}{(x^2+x-2)^2+3}}{\left[\dfrac{x^2+x-2}{(x^2+x-2)^2}\right]^2+3}$

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2. Find the domain of the following functions.

a) $f(x) = \dfrac{1}{x^2+x-12}$

b) $g(x) = \dfrac{3x+2}{x^2+2x-2}$

c) $h(x) = \dfrac{1}{x^2-1} + \dfrac{x}{x^2-4}$

d) $i(x) = \sqrt{3-x} + \sqrt{x+4}$

Solution:

a) Exclude the values where the denominator is 0, i.e. the solutions of $x^2+x-12=0$. To solve this equation, factor: $x^2+x-12 = (x-3)(x+4) = 0$, so $x=3$ or $x=-4$. Therefore the domain of f is $(-\infty-4)\cup(-4,3)\cup(3,\infty)$.

b) Exclude the values where the denominator is 0, i.e. the solutions of $x^2+2x-2=0$. To solve this equation, use the quadratic formula:

$$x = \frac{-2\pm\sqrt{2^2-4\cdot 1\cdot(-2)}}{2\cdot 1} = \frac{-2\pm\sqrt{12}}{2} = -1\pm\sqrt{3}.$$

Therefore the numbers $-1+\sqrt{3}$ and $-1-\sqrt{3}$ should be excluded from the domain of g. Therefore the domain of g is $(-\infty,-1-\sqrt{3})\cup(-1-\sqrt{3},-1+\sqrt{3})\cup(-1+\sqrt{3},\infty)$.

c) Exclude the values of x where either of the denominators is 0, i.e. where $x^2-1=0$ or where $x^2-4=0$. The solutions to these equations are $-4,-1,1$ and 4. Therefore the domain of h is $(-\infty,-4)\cup(-4,-1)\cup(-1,1)\cup(1,4)\cup(4,\infty)$.

d) The values of x in the domain of i are those that are both in the domain of $\sqrt{3-x}$ and in the domain of $\sqrt{x+4}$. These are the x such that

$$3-x\geq 0 \quad \textbf{and} \quad x+4\geq 0,$$

The solution of these inequalities is $x\leq 3$ and $x\geq -4$, respectively. The domain of i is therefore $[-4,3]$.

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3. Given $f(x) = 2x+1$ and $g(x) = x^2+3$, find and simplify:

a) $(f\circ g)(3)$

b) $(f\circ g)(x)$

c) $(g\circ f)(x)$

d) $(f\circ g\circ f)(x)$

Solution:

a) $(f \circ g)(3) = f(g(3)) = f(12) = 25$

b) $(f \circ g)(x) = f(g(x)) = f(x^2 + 3) = 2(x^2 + 3) + 1 = 2x^2 + 7$

c) $(g \circ f)(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 + 3 = 4x^2 + 4x + 4 = 4(x^2 + x + 1)$

d) $(f \circ g \circ f)(x) = f(g(f(x))) = f(g(2x + 1)) = f((2x + 1)^2 + 3) = 2((2x + 1)^2 + 3) + 1 = 8x^2 + 8x + 9$

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4. Given $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{2}{x-1}$, find and simplify

a) $(f \circ g)(3)$ **b)** $(f \circ g)(x)$ **c)** $(g \circ f)(x)$ **d)** $(f \circ g \circ f)(x)$

Solution:

a) $(f \circ g)(3) = f(g(3)) = f(1) = 1$

b) $(f \circ g)(x) = f(g(x)) = f(\frac{2}{x-1}) = \dfrac{1}{\frac{2}{x-1}} = \frac{x-1}{2}.$

c) $(g \circ f)(x) = g(f(x)) = g(\frac{1}{x}) = \dfrac{2}{\frac{1}{x} - 1} = \frac{2x}{1-x}$

d) $(f \circ g \circ f)(x) = f(g(f(x))) = f(g(\frac{1}{x})) = f(\dfrac{2}{\frac{1}{x} - 1}) = f(\frac{2x}{1-x}) = \dfrac{1}{\frac{2x}{1-x}} = \frac{1-x}{2x}$

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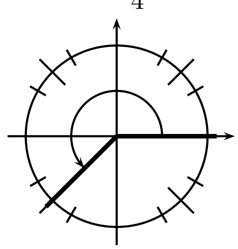
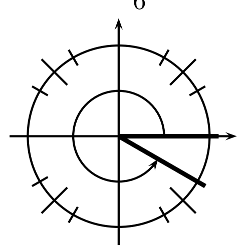
5. Draw the following angles in standard position in the circles provided.

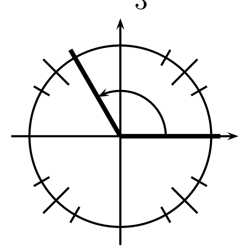
a) $\frac{5\pi}{4}$



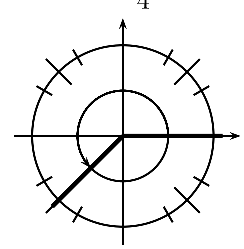
b) $\frac{11\pi}{6}$



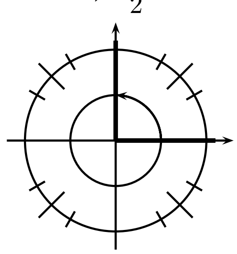
c) $\frac{2\pi}{3}$



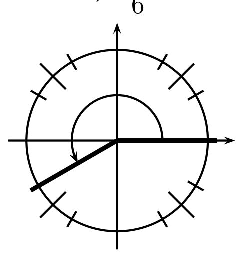
d) $\frac{13\pi}{4}$



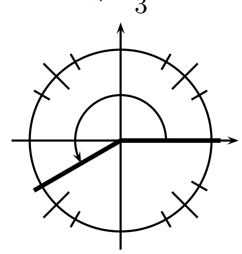
e) $\frac{5\pi}{2}$



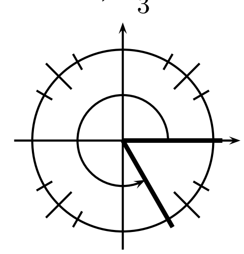
f) $\frac{7\pi}{6}$



g) $\frac{7\pi}{3}$



h) $\frac{5\pi}{3}$



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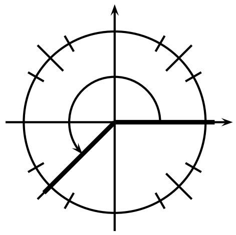
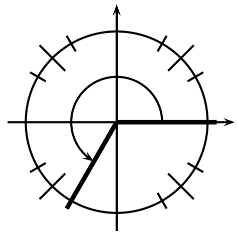
6. Write, in the space provided, the value IN RADIANS of the angles given in the following pictures.

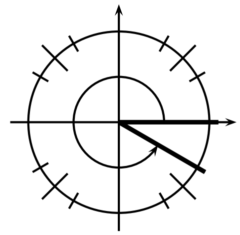
a) ....



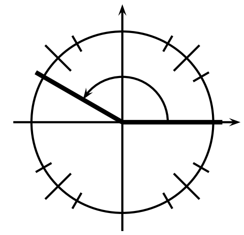
b) ....



c) ....

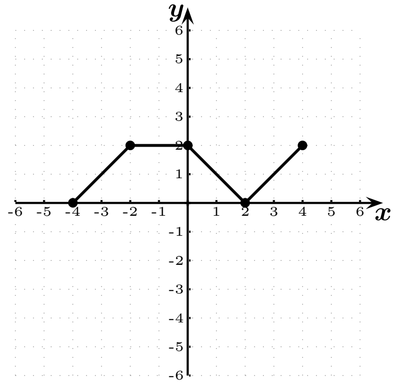


d) ....

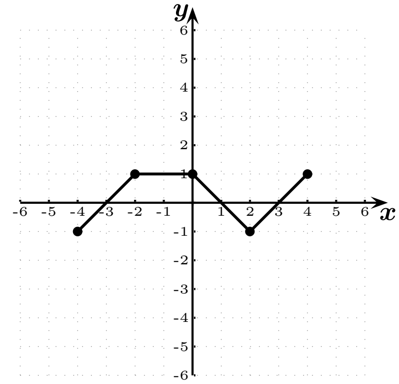


7. Use the graph of $y = f(x)$ to graph each function g. You can use the axes provided in this sheet or do the graphs **in graph paper**.

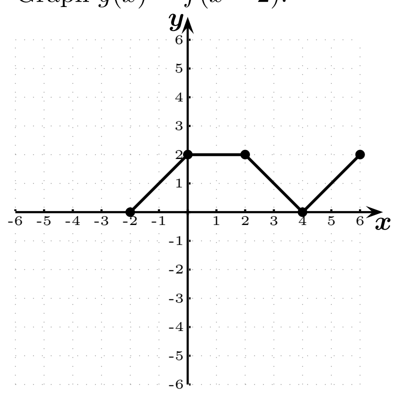
This is the given original graph of f.



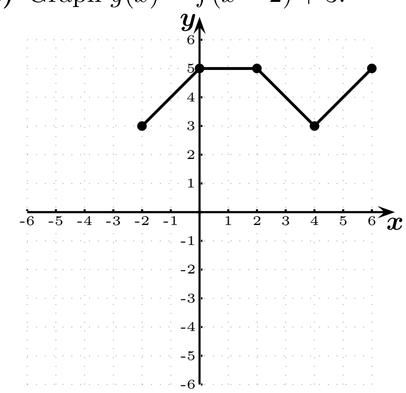
a) Graph $g(x) = f(x) - 1$.



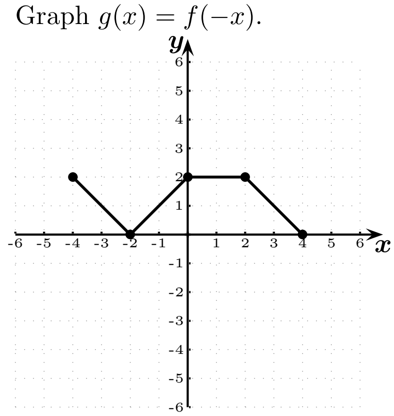
b) Graph $g(x) = f(x - 2)$.



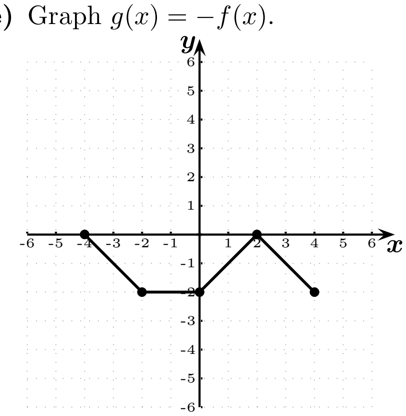
c) Graph $g(x) = f(x - 2) + 3$.



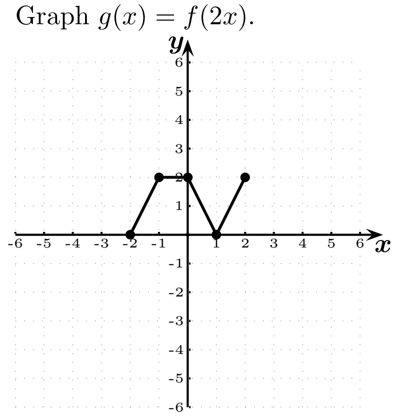
d) Graph $g(x) = f(-x)$.



e) Graph $g(x) = -f(x)$.



f) Graph $g(x) = f(2x)$.



g) Graph $g(x) = 2f(x)$.