MTH30

Review sheet for Midterm 3

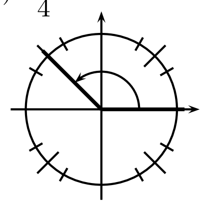
Professor Luis Fernandez

NOTE: Solutions available upon request. If you want the solution, please write me an email and I will send it to you. But only look at the solution *after* you have done the exercises.

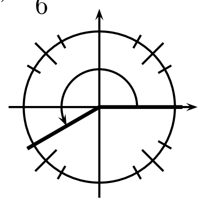
Solutions

1. Draw the following angles in standard position in the circles provided.

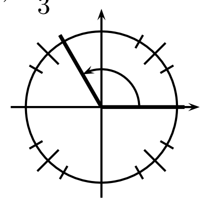
(a) $\frac{3\pi}{4}$



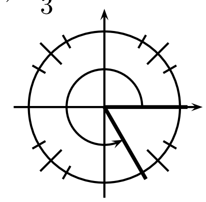
(b) $\frac{7\pi}{6}$



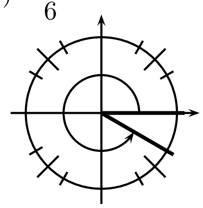
(c) $\frac{2\pi}{3}$



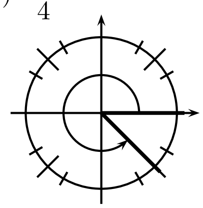
(d) $\frac{5\pi}{3}$



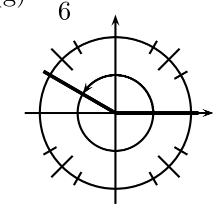
(e) $\frac{11\pi}{6}$



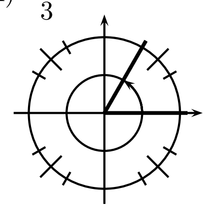
(f) $\frac{7\pi}{4}$



(g) $\frac{17\pi}{6}$

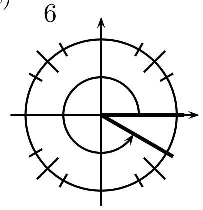


(h) $\frac{7\pi}{3}$

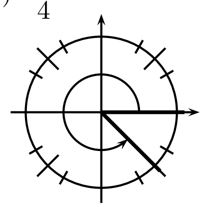


2. Write, in the space provided, the value IN RADIANS of the angles given in the following pictures.

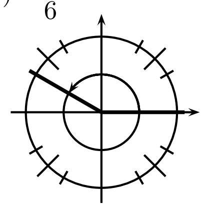
(a) $\frac{11\pi}{6}$



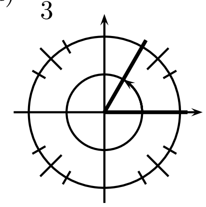
(b) $\frac{7\pi}{4}$



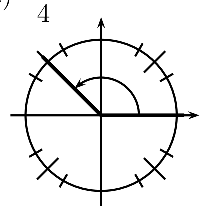
(c) $\frac{17\pi}{6}$



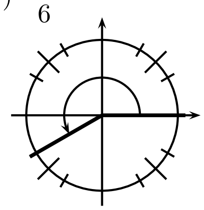
(d) $\frac{7\pi}{3}$



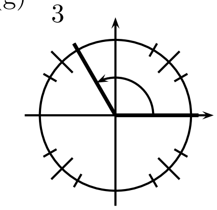
(e) $\frac{3\pi}{4}$



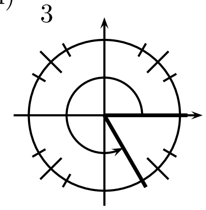
(f) $\frac{7\pi}{6}$



(g) $\frac{2\pi}{3}$



(h) $\frac{5\pi}{3}$



3. Find an angle (in degrees) between 0° and 360° that is coterminal with the following angles:

(a) 425° (b) 1225° (c) −560°

Solution:

(a) 65° (b) 145° (c) 160°

4. Find an angle (in radians) between 0 and 2π that is coterminal with the following angles:

(a) 11π (b) $\frac{11\pi}{2}$ (c) $\frac{19\pi}{4}$ (d) $\frac{-9\pi}{2}$

Solution:

(a) π (b) $\frac{3\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

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5. Find the reference angle of the following angles (in degrees).

(a) $115°$ (b) $267°$ (c) $333°$ (d) $-100°$

Solution:

(a) $65°$ (b) $87°$ $27°$ (c) $80°$

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6. Find the reference angle of the following angles (in radians).

(a) $\frac{7\pi}{6}$ $\frac{5\pi}{4}$ $\frac{2\pi}{3}$ $-\frac{5\pi}{6}$

Solution:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

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7. Convert from radians to degrees. **Solution:**

(a) $\frac{7\pi}{6} = 210°$ (b) $\frac{5\pi}{4} = 225°$ (c) $\frac{5\pi}{3} = 300°$ (d) $-\frac{11\pi}{6} = -330°$

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8. Convert from degrees to radians.

Solution:

(a) $150° = \frac{5\pi}{6}$ (b) $240° = \frac{4\pi}{3}$ (c) $315° = \frac{7\pi}{4}$ (d) $-150° = -\frac{5\pi}{6}$

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9. Given that $\cos\alpha = -\frac{4}{5}$, and that α is in Quadrant II, find the exact value of $\sin\alpha$, $\tan\alpha$, $\sec\alpha$, $\csc\alpha$, $\cot\alpha$.

Solution: $\sin\alpha = \frac{3}{5}$, $\tan\alpha = -\frac{3}{4}$, $\cot\alpha = -\frac{4}{3}$, $\sec\alpha = -\frac{5}{4}$, $\csc\alpha = \frac{5}{3}$.

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10. Given that $\tan\alpha = -\frac{2}{3}$, and that α is in Quadrant IV, find the exact value of $\sin\alpha$, $\cos\alpha$, $\sec\alpha$, $\csc\alpha$, $\cot\alpha$.

Solution: $\sin\alpha = -\frac{2}{\sqrt{13}}$, $\cos\alpha = \frac{3}{\sqrt{13}}$, $\cot\alpha = -\frac{3}{2}$, $\sec\alpha = \frac{\sqrt{13}}{3}$, $\csc\alpha = -\frac{\sqrt{13}}{2}$.

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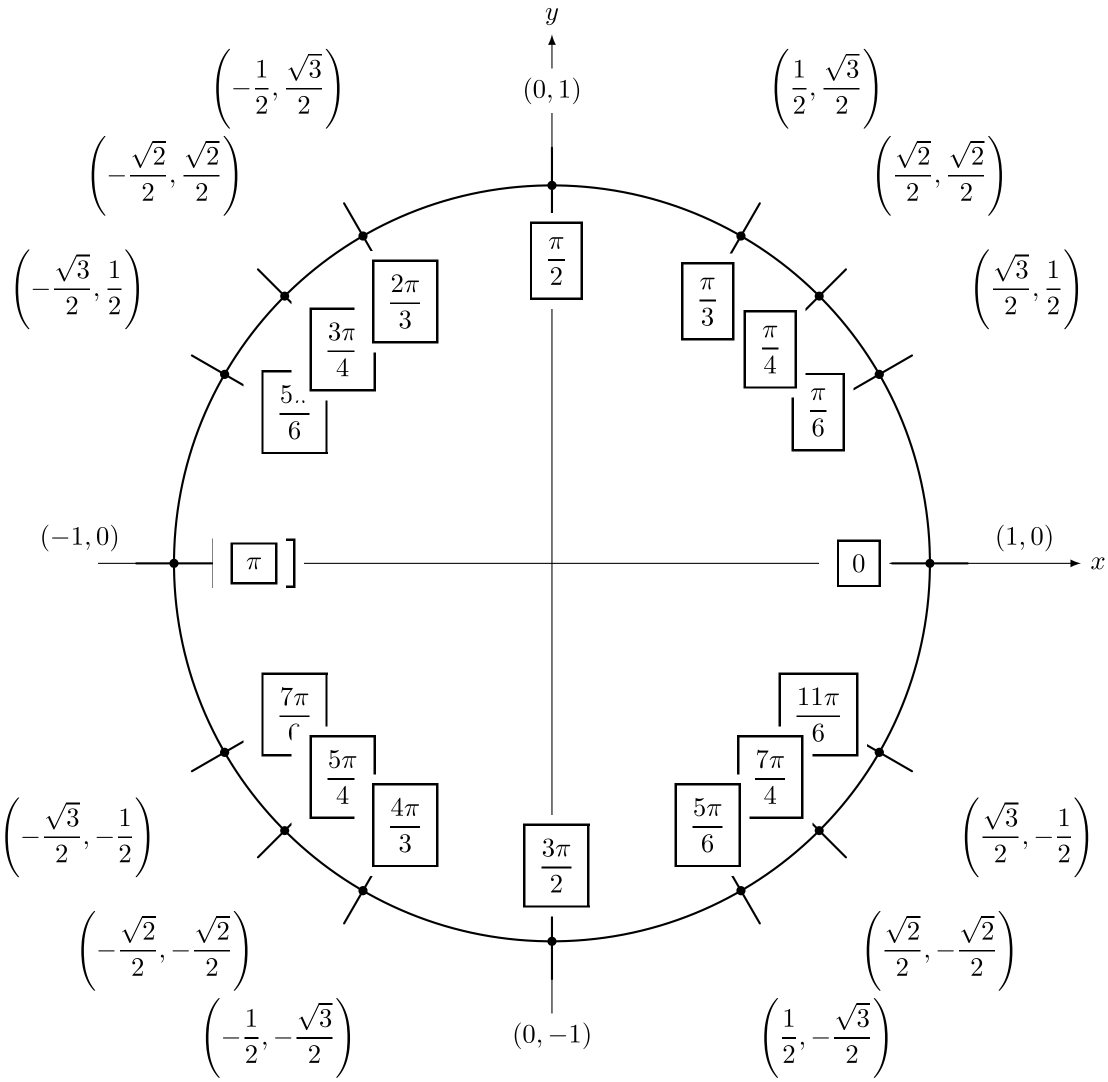
11. Fill in the angles, **in radians**, inside the boxes. Then fill in the coordinates of the points marked in the circle. And remember that the sine of an angle is the y coordinate, and the cosine is the x coordinate.



12. For the following sinusoidal functions, find the amplitude, the period, and the phase shift.
For (a) and (c), graph a full period of the function.

(a) $f(x) = 2\sin\left(3x - \frac{3\pi}{2}\right)$

(b) $f(x) = -6\sin\left(4x - \frac{\pi}{2}\right)$

(c) $f(x) = \frac{4}{3}\cos\left(2\pi x - \frac{\pi}{2}\right)$

(d) $f(x) = 4\sin\left(3x + \frac{3\pi}{4}\right)$

Solution:

(a) Amplitude: 2. Period: $\frac{2\pi}{3}$. Phase shift: $\frac{\pi}{2}$.

(b) Amplitude: 6. Period: $\frac{\pi}{2}$. Phase shift: $\frac{\pi}{8}$.

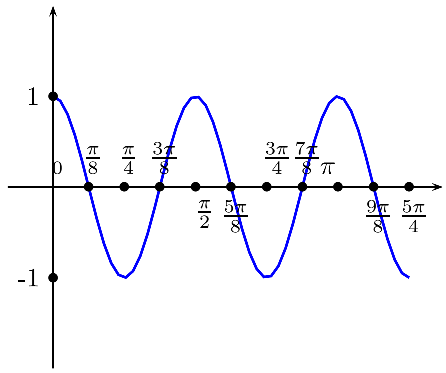
(c) Amplitude: $\frac{4}{3}$. Period: 1. Phase shift: $\frac{1}{4}$.

(d) Amplitude: 4. Period: $\frac{2\pi}{3}$. Phase shift: $\frac{\pi}{4}$.

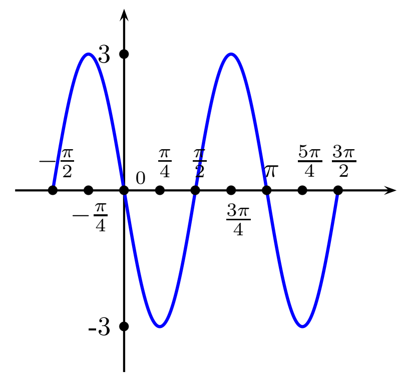
Please use a computer to check the graphs (for example, Desmos).

13. The following are the graphs of functions of the form $f(x) = A\sin(Bx - C)$, with $A > 0$. Find the amplitude, the period, and the phase shift. Then use this information to find the values of A, B and C (recall that A will be equal to the amplitude, that the period is $2\pi/B$, and that the phase shift equals B/C).

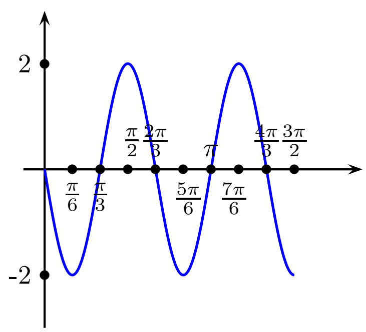
(a)



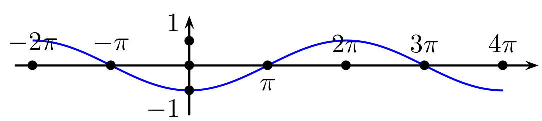
(b)



(c)



(d)



Solution:

(a) Amplitude: 1. Period: $\dfrac{\pi}{2}$. Phase shift: $\dfrac{3\pi}{8}$. $A = 1$, $B = 4$, $C = \dfrac{3\pi}{2}$, so $f(x) = \sin(4x - \frac{3\pi}{2})$.

(b) Amplitude: 3. Period: π. Phase shift: $\dfrac{\pi}{2}$. $A = 3$, $B = 2$, $C = \pi$, so $f(x) = 3\sin(2x - \pi)$.

(c) Amplitude: 2. Period: $\dfrac{2\pi}{3}$. Phase shift: $\dfrac{\pi}{3}$. $A = 2$, $B = 3$, $C = \pi$, so $f(x) = 2\sin(3x - \pi)$.

(d) Amplitude: 1. Period: 4π. Phase shift: π. $A = 1$, $B = \dfrac{1}{2}$, $C = \dfrac{\pi}{2}$, so $f(x) = \sin\left(\dfrac{x}{2} - \dfrac{\pi}{2}\right)$.

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14. Prove the following trigonometric identities.

(a) $\cos x - \cos^3 x = \cos x \sin^2 x$

(b) $\cos x\,(\tan x - \sec x) = \sin x - 1$

(c) $\sec^2 x(1 - \cos^2 x) = \tan^2 x$

(d) $\sin x\,(\cot x + \csc x) = \cos x + 1$

(e) $\cos^2 x(1 + \tan^2 x) = 1$

(f) $\sin x\ \tan x = \sec x - \cos x$.

(g) $\sec x - \cos x = \tan x\ \sin x$.

(h) $\sec x\ \csc x = \tan x + \cot x$.

(i) $\dfrac{\cos x\ \sec x}{\cot x} = \tan x$

(j) $\tan x = \dfrac{\cos x\ \sec x}{\cot x}$

Solution:

(a) $\cos x\,(\tan x - \sec x) = \cos x\left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right) = \frac{\sin x \cos x}{\cos x} - \frac{\cos x}{\cos x} = \sin x - 1$

(b) $\cos x - \cos^3 x = \cos x(1-\cos^2 x) = \cos x \sin^2 x$

(c) $\sec^2 x(1-\cos^2 x) = \frac{1}{\cos^2 x} \cdot \sin^2 x = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$

(d) $\sin x\,(\cot x + \csc x) = \sin x\left(\frac{\cos x}{\sin x} + \frac{1}{\sin x}\right) = \frac{\sin x \cos x}{\sin x} + \frac{\sin x}{\sin x} = \cos x + 1$

(e) $\cos^2 x(1+\tan^2 x) = \cos^2 x\left(1 + \frac{\sin^2 x}{\cos^2 x}\right) = \cos^2 x + \frac{\sin^2 x \cos^2 x}{\cos^2 x} = \cos^2 x + \sin^2 x = 1$

(f) Simplify the right hand side:
$\sec x - \cos x = \frac{1}{\cos x} - \cos x = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$
On the other hand, $\sin x\ \tan x = \sin x \cdot \frac{\sin x}{\cos x} = \frac{\sin^2 x}{\cos x}$.
Therefore, both sides are equal to $\frac{\sin^2 x}{\cos x}$, so they must be equal to each other.

(g) $\sec x - \cos x = \frac{1}{\cos x} - \cos x = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \sin x \tan x$

(h) simplify the right hand side:
$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \frac{1}{\sin x} + \frac{1}{\cos x} = \sec x + \csc x$

(i) $\frac{\cos x \sec x}{\cot x} = \frac{\cos x \frac{1}{\cos x}}{\frac{\cos x}{\sin x}} = \frac{\cos x}{\cos x} \cdot \frac{\sin x}{\cos x} = \tan x$

(j) Same as the previous one.

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15. Solve the following equations, for x in the interval $0 \le x < 2\pi$.

(a) $3\sin x - 1 = \sin x$ (b) $3\cos x + 1 = \cos x$ (c) $3\sin x + 1 = \sin x$
(d) $3\sin x + \sqrt{3} = \sin x$ (e) $3\cos x - 1 = \cos x$ (f) $(\tan x)^2 = 3$

Solution:

(a) $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

(b) $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$

(c) $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$

(d) $x = \frac{4\pi}{3}$ and $x = \frac{5\pi}{3}$

(e) $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

(f) $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$,$x = \frac{4\pi}{3}$, and $x = \frac{5\pi}{3}$