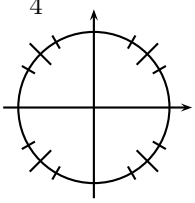
MTH30

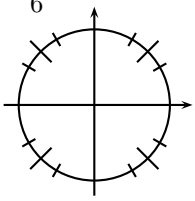
Review sheet for Midterm 3

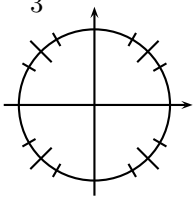
Professor Luis Fernandez

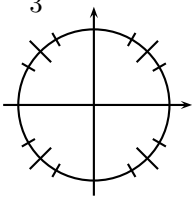
NOTE: Solutions available upon request. If you want the solution, please write me an email and I will send it to you. But only look at the solution *after* you have done the exercises.

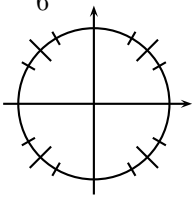
1. Draw the following angles in standard position in the circles provided.

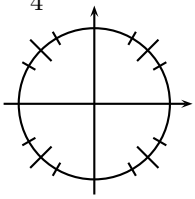
(a) $\frac{3\pi}{4}$ 

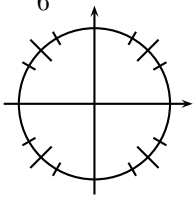
(b) $\frac{7\pi}{6}$ 

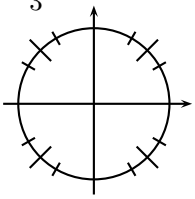
(c) $\frac{2\pi}{3}$ 

(d) $\frac{5\pi}{3}$ 

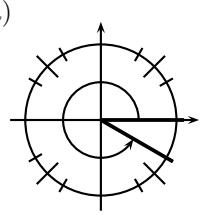
(e) $\frac{11\pi}{6}$ 

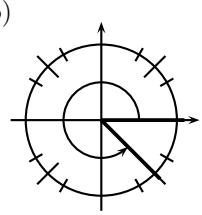
(f) $\frac{7\pi}{4}$ 

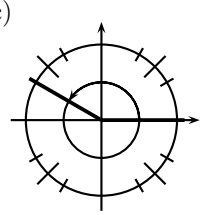
(g) $\frac{17\pi}{6}$ 

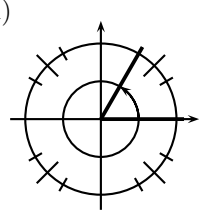
(h) $\frac{7\pi}{3}$ 

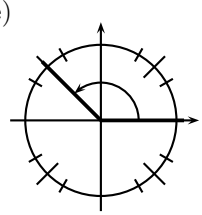
2. Write, in the space provided, the value IN RADIANS of the angles given in the following pictures.

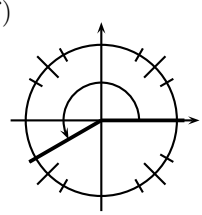
(a) 

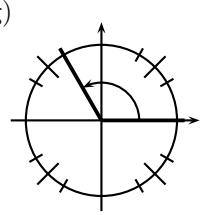
(b) 

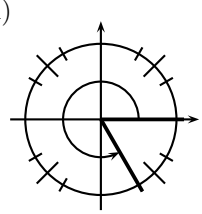
(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

3. Find an angle (in degrees) between 0° and 360° that is coterminal with the following angles:

(a) 425° (b) 1225° (c) -560°

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4. Find an angle (in radians) between 0 and 2π that is coterminal with the following angles:

(a) 11π (b) $\frac{11\pi}{2}$ (c) $\frac{19\pi}{4}$ (d) $\frac{-9\pi}{2}$

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5. Find the reference angle of the following angles (in degrees).

(a) 115° (b) 267° (c) 333° (d) -100°

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6. Find the reference angle of the following angles (in radians).

(a) $\frac{7\pi}{6}$ $\frac{5\pi}{4}$ $\frac{2\pi}{3}$ $-\frac{5\pi}{6}$

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7. Convert from radians to degrees.

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{4}$ (c) $\frac{5\pi}{3}$ (d) $-\frac{11\pi}{6}$

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8. Convert from degrees to radians.

(a) 150° (b) 240° (c) 315° (d) -150°

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9. Given that $\cos\alpha = -\frac{4}{5}$, and that α is in Quadrant II, find the exact value of $\sin\alpha$, $\tan\alpha$, $\sec\alpha$, $\csc\alpha$, $\cot\alpha$.

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10. Given that $\tan\alpha = -\frac{2}{3}$, and that α is in Quadrant IV, find the exact value of $\sin\alpha$, $\cos\alpha$, $\sec\alpha$, $\csc\alpha$, $\cot\alpha$.

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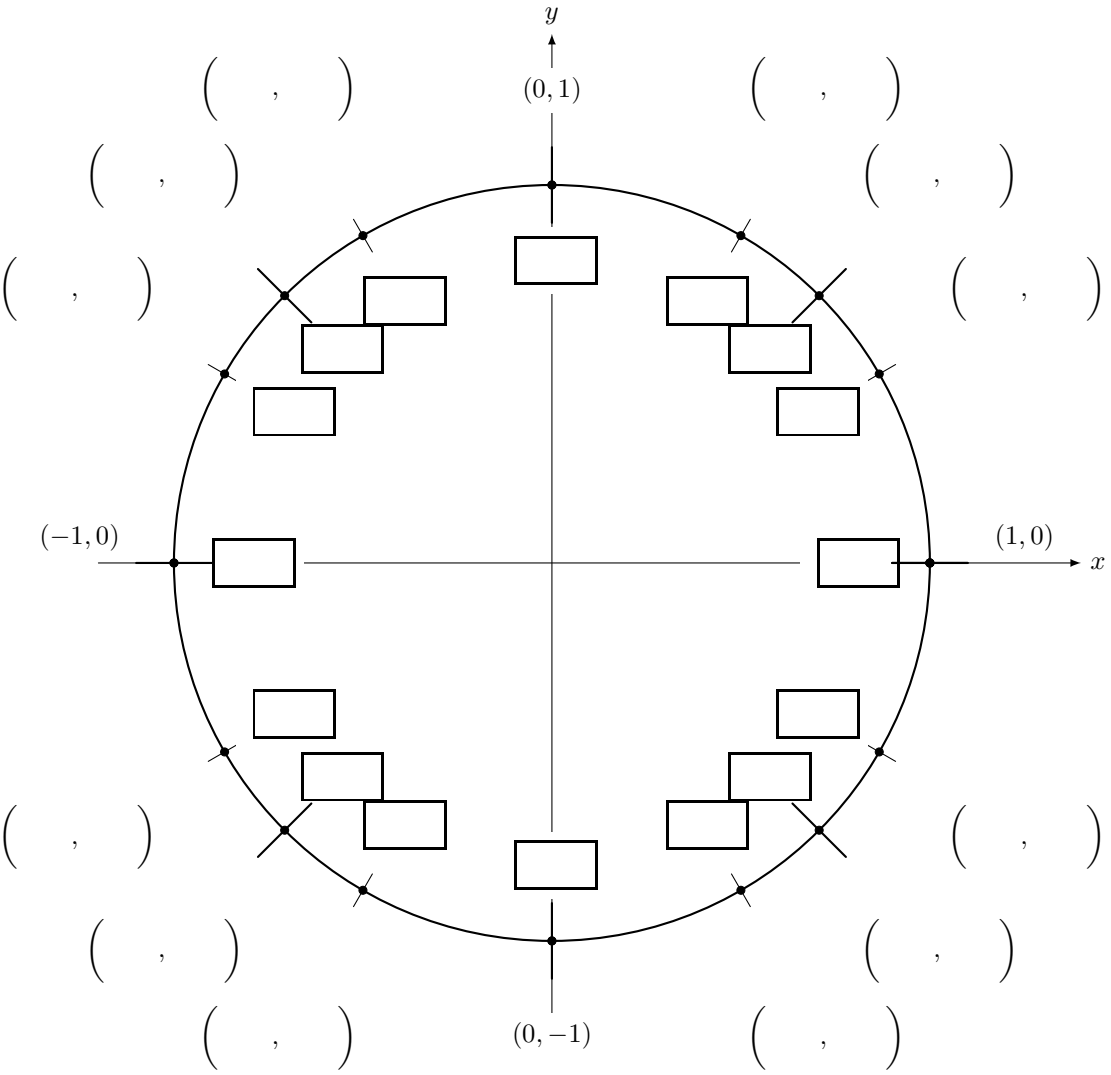
11. Fill in the angles, **in radians**, inside the boxes. Then fill in the coordinates of the points marked in the circle. And remember that the sine of an angle is the y coordinate, and the cosine is the x coordinate.



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12. For the following sinusoidal functions, find the amplitude, the period, and the phase shift.
For (a) and (c), graph a full period of the function.

(a) $f(x) = 2\sin\left(3x - \dfrac{3\pi}{2}\right)$

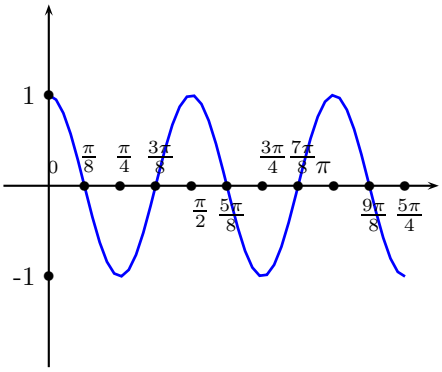
(b) $f(x) = -6\sin\left(4x - \dfrac{\pi}{2}\right)$

(c) $f(x) = \dfrac{4}{3}\cos\left(2\pi x - \dfrac{\pi}{2}\right)$

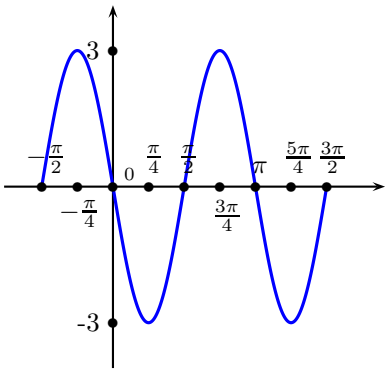
(d) $f(x) = 4\sin\left(3x + \dfrac{3\pi}{4}\right)$

13. The following are the graphs of functions of the form $f(x) = A\sin(Bx - C)$, with $A > 0$. Find the amplitude, the period, and the phase shift. Then use this information to find the values of A, B and C (recall that A will be equal to the amplitude, that the period is $2\pi/B$, and that the phase shift equals B/C).

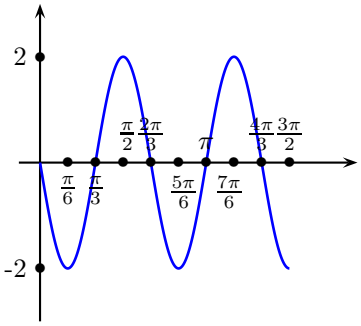
(a)



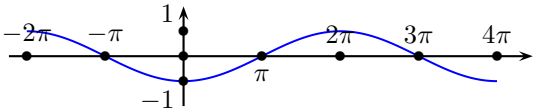
(b)



(c)



(d)



14. Prove the following trigonometric identities.

(a) $\cos x - \cos^3 x = \cos x \sin^2 x$

(b) $\cos x\,(\tan x - \sec x) = \sin x - 1$

(c) $\sec^2 x(1 - \cos^2 x) = \tan^2 x$

(d) $\sin x\,(\cot x + \csc x) = \cos x + 1$

(e) $\cos^2 x(1 + \tan^2 x) = 1$

(f) $\sin x\,\tan x = \sec x - \cos x.$

(g) $\sec x - \cos x = \tan x\,\sin x.$

(h) $\sec x\,\csc x = \tan x + \cot x.$

(i) $\dfrac{\cos x\,\sec x}{\cot x} = \tan x$

(j) $\tan x = \dfrac{\cos x\,\sec x}{\cot x}$

15. Solve the following equations, for x in the interval $0 \le x < 2\pi$.

(a) $3\sin x - 1 = \sin x$

(b) $3\cos x + 1 = \cos x$

(c) $3\sin x + 1 = \sin x$

(d) $3\sin x + \sqrt{3} = \sin x$

(e) $3\cos x - 1 = \cos x$

(f) $(\tan x)^2 = 3$