MTH30

Review sheet for Midterm 2

Professor Luis Fernandez

1. For each of the following rational functions f

   A. $f(x) = \dfrac{x+1}{x-2}$ B. $f(x) = \dfrac{x^2+2x-3}{x^2-2x-3}$ C. $f(x) = \dfrac{x^2-9}{x^2-x-2}$ D. $f(x) = \dfrac{2-x}{x^2+x-2}$
   E. $f(x) = \dfrac{x^2}{x^2+1}$

   (a) Factor numerator and denominator and simplify if possible.
   (b) Find the x and y intercepts of the graph of $y = f(x)$ if they exist.
   (c) Find any vertical or horizontal asymptotes.
   (d) Use the above information to sketch a graph of $y = f(x)$.

2. For the following functions, find $f(-3)$, $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, $f(2)$, $f(3)$. Then plot the points you got and sketch their graph.

   A. $f(x) = 2^x$ B. $f(x) = 3^x$ C. $f(x) = (\frac{1}{2})^x$ D. $f(x) = 2^{-x}$

3. Convert the following from exponential form to logarithmic form.

   A. $e^x = 5$. B. $4^{x+3} = 7$ C. $\left(\dfrac{1}{3}\right)^{2y+1} = x-3$ D. $10^{x+2} = 14$.

4. Convert the following from exponential form to logarithmic form.

   A. $\operatorname{Ln} y = 7$. B. $\log_5(y+3) = x+7$ C. $\log_{\frac{1}{3}}(2y+1) = 5$ D. $\log(x+2) = 12$.

5. Expand

   (a) $\log_7\left(x^4y^3\right)$
   (b) $\log_3 \dfrac{x^4y^3}{z^2w^8}$
   (c) $\log\left(x^4y^3\right)^5$
   (d) $\log\sqrt[4]{\dfrac{10x^2y^3}{5z}}$

6. Condense

   (a) $3\log x + 7\log y$
   (b) $4\log_4 x - 5\log_4 y + \log_4 z - 3\log_4 w$
   (c) $\dfrac{1}{2}\operatorname{Ln} x - \dfrac{2}{6}\operatorname{Ln} y + \dfrac{3}{4}\operatorname{Ln} z$
   (d) $\dfrac{1}{5}(2\log x - \frac{1}{2}\log y + \frac{2}{3}\log z)$

7. Evaluate the following expressions. Give exact values whenever possible:

   (a) $\log_2 \dfrac{1}{64}$
   (b) $\log_9 \dfrac{\sqrt{3}}{3}$
   (c) $\log_b x^3y$, given that $\log_b x = 2$ and $\log_b y = 36$
   (d) e^{x-y} given that $e^x = 3$ and $e^y = 4$

(e) $\log_a \left(\frac{x}{y}\right)$ given that $\log_a(x) = 12$ and $\log_a(y) = 4$

(f) $\ln e^{\sqrt{2}}$

(g) $\log 1000$

(h) $\log_7 31$, rounded to the nearest hundredth

(i) $e^{\text{Ln}\,5}$

(j) $\log_7 7^{124}$

8. Write the following logarithms in the indicated base. Simplify what you can.

(a) $\log_5 7$, in base 7.

(b) $\log_8 4$, in base 2.

(c) $\log_6 10$, in base e.

9. Solve the following equations. If the answer is not an exact numbers, leave it expressed as a logarithm.

(a) $7^{x+2} = 49$

(b) $4^{x+3} = 8^{2x-4}$

(c) $e^x = 2$

(d) $3^{x+5} = 9 \cdot 3^{x+2}$

(e) $\log_2 x - \log_2(x-1) = 1$

(f) $\log_3 x - 2 = \log_3 4$

(g) $\log_5(x+2) + \log_5(x+3) = \log_5(1-x)$

(h) $4 + \log_2(9x) = 2$

Solutions

1. (a) A. $\dfrac{x+1}{x-2}$ B. $\dfrac{(x+3)(x-1)}{(x+1)(x-3)}$ C. $\dfrac{(x+3)(x-3)}{(x-2)(x+1)}$ D. $\dfrac{2-x}{(x+2)(x-1)}$ E. $\dfrac{x^2}{x^2+1}$

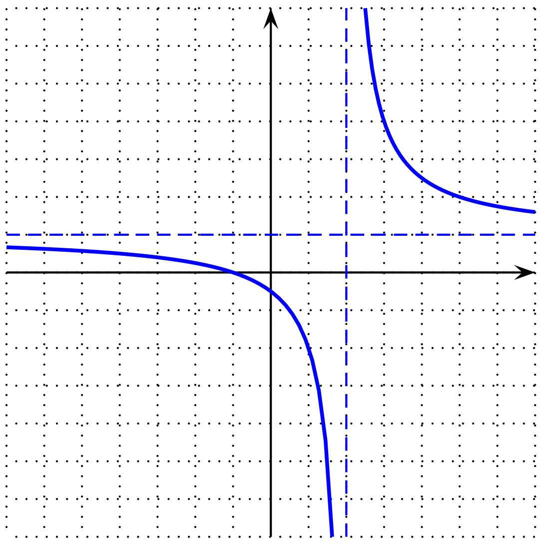
   (b) x-intercepts: A. $x = 1$ B. $x = -3$, $x = 1$ C. $x = -3$, $x = 3$ D. $x = 2$ E. $x = 0$

   y-intercepts: A. $y = -\dfrac{1}{2}$ B. $y = 1$ C. $y = \dfrac{9}{2}$ D. $y = -1$ E. $y = 0$

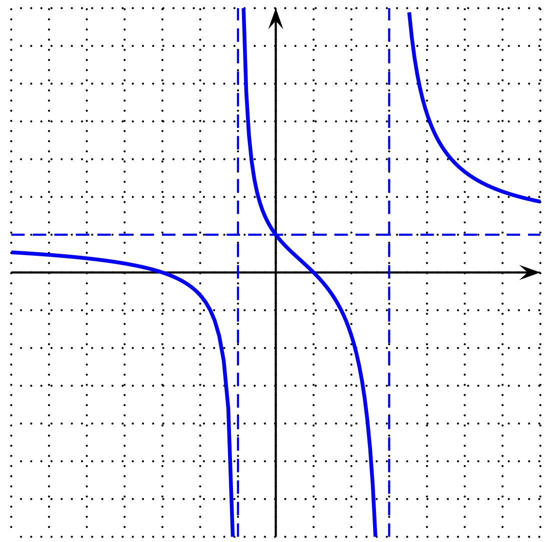
   (c) Vertical asymptotes A. $x = 2$ B. $x = 3$, $x = -1$ C. $x = 2$, $x = -1$ D. $x = -2$, $x = 1$ E. None

   (d) Horizontal asymptotes A. $y = 1$ B. $y = 1$ C. $y = 1$ D. $y = 0$ E. $y = 1$

   (e) The graphs:



A



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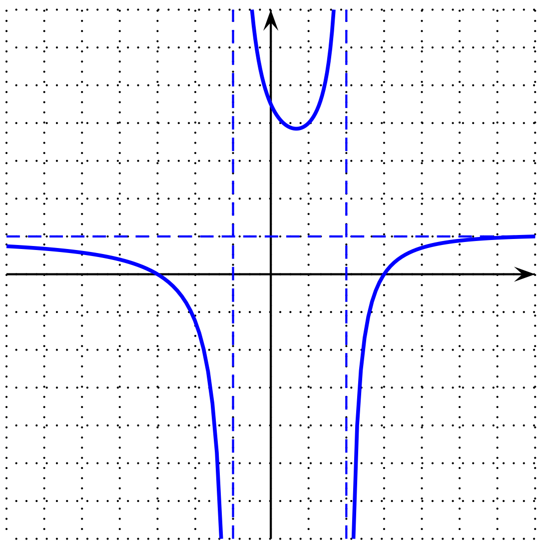

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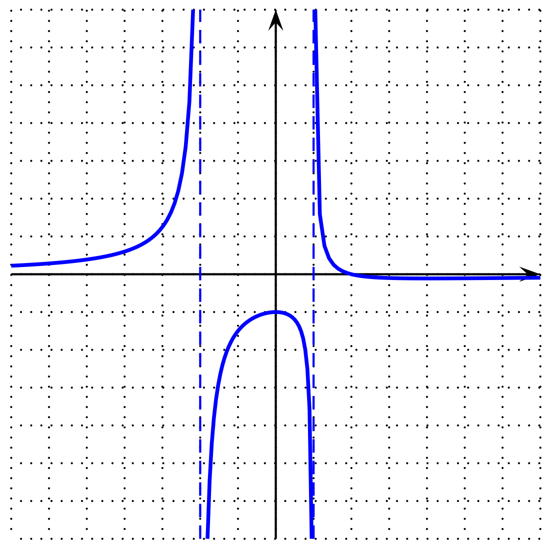
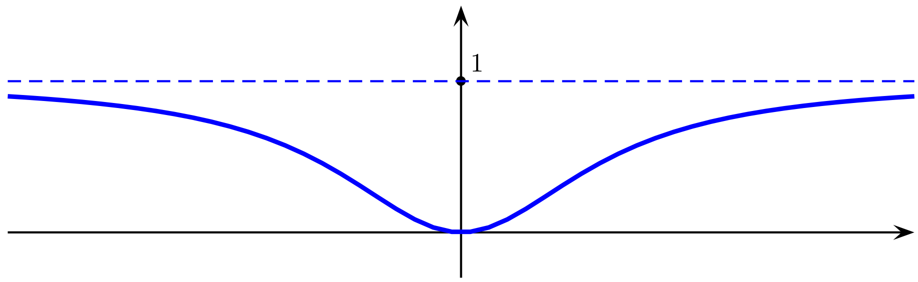


D



E

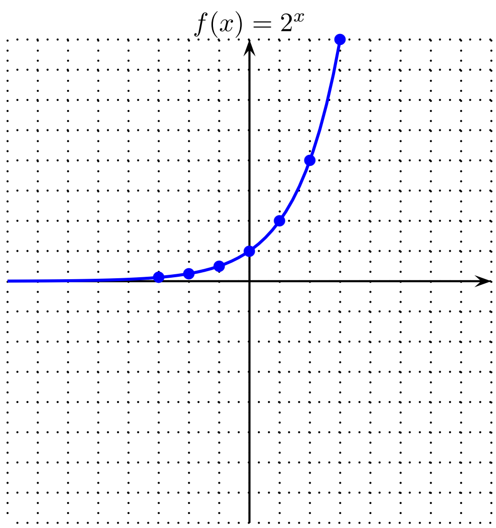
2. A. $f(-3) = \frac{1}{8}$, $f(-2) = \frac{1}{4}$, $f(-1) = \frac{1}{2}$, $f(0) = 1$, $f(1) = 2$, $f(2) = 4$, $f(3) = 8$.

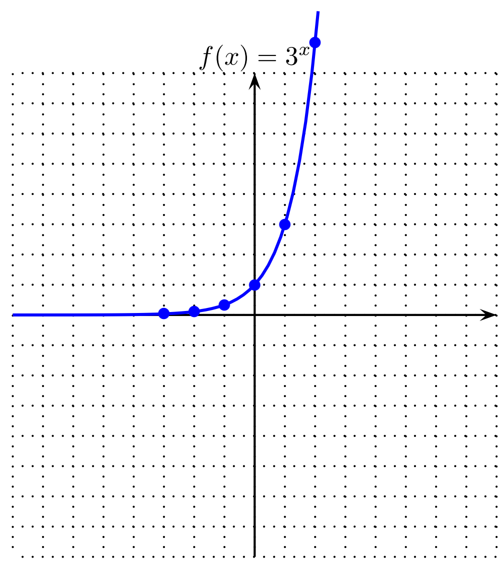
B. $f(-3) = \frac{1}{27}$, $f(-2) = \frac{1}{9}$, $f(-1) = \frac{1}{3}$, $f(0) = 1$, $f(1) = 3$, $f(2) = 9$, $f(3) = 27$.

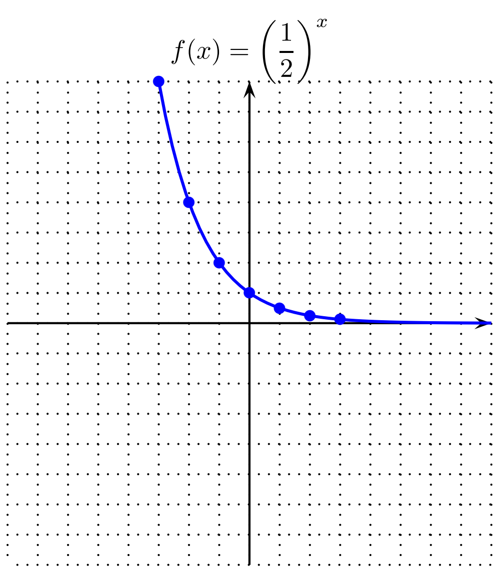
C. $f(-3) = 8$, $f(-2) = 4$, $f(-1) = 2$, $f(0) = 1$, $f(1) = \frac{1}{2}$, $f(2) = \frac{1}{4}$, $f(3) = \frac{1}{8}$.

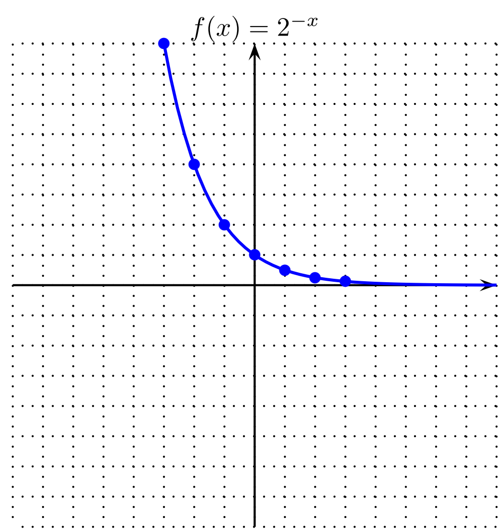
D. Same as previous one, because $\left(\frac{1}{2}\right)^x = 2^{-x}$.

The graphs:









3. (a) $\log_7\left(x^4y^3\right) = 4\log_7 x + 3\log_7 y$
   (b) $\log_3 \dfrac{x^4y^3}{z^2w^8} = 4\log_3 x + 3\log_3 y - 2\log_3 z - 8\log_3 w$
   (c) $\log\left(x^4y^3\right)^5 = 5(4\log x + 3\log y)$
   (d) $\log\sqrt[4]{\dfrac{10x^2y^3}{5\sqrt{z}}} = \dfrac{1}{4}(\log 10 + 2\log x + 3\log y - \log 5 - \frac{1}{2}\log z)$
4. (a) $3\log x + 7\log y = \log(x^3y^7)$
   (b) $4\log_4 x - 5\log_4 y + \log_4 z - 3\log_4 w = \log_4\left(\dfrac{x^4z}{y^5w^3}\right)$
   (c) $\dfrac{1}{2}\operatorname{Ln} x - \dfrac{5}{6}\operatorname{Ln} y + \dfrac{3}{4}\operatorname{Ln} z = \operatorname{Ln}\left(\dfrac{x^{\frac{1}{2}}z^{\frac{3}{4}}}{y^{\frac{5}{6}}}\right)$ or $\operatorname{Ln}\left(\dfrac{\sqrt{x}\sqrt[4]{z^3}}{\sqrt[6]{y^5}}\right)$
   (d) $\dfrac{1}{5}(2\log x - \frac{1}{2}\log y + \frac{2}{3}\log z) = \log\left(\sqrt[5]{\dfrac{x^2\sqrt[3]{z^2}}{\sqrt{y}}}\right)$
5. (a) $\log_5 7$, in base 7 is $\dfrac{\log_7 7}{\log_7 5} = \dfrac{1}{\log_7 5}$
   (b) $\log_8 4$, in base 2 is $\dfrac{\log_2 4}{\log_2 8} = \dfrac{2}{3}$
   (c) $\log_6 10$, in base e is $\dfrac{\operatorname{Ln} 10}{\operatorname{Ln} 6}$
6. (a) -6
   (b) $-\dfrac{1}{4}$
   (c) 42
   (d) $\dfrac{3}{4}$
   (e) 8
   (f) $\sqrt{2}$
   (g) 3
   (h) 1.76
   (i) 5
   (j) 124
7. Solve the following equations. If the answer is not an exact numbers, leave it expressed as a logarithm.
   (a) $7^{x+2} = 49$: Write it as $7^{x+2} = 7^2$. Solution is $x = 0$.
   (b) $4^{x+3} = 8^{2x-4}$: Write it as $2^{2(x+3)} = 2^{3(2x-4)}$. Solution is $x = \dfrac{9}{2}$.
   (c) $e^x = 2$. Take Ln of both sides. It gives $x = \operatorname{Ln} 2$.
   (d) $3^{x+5} = 9 \cdot 3^{2x+2}$. Take $\log_3$ of both sides to get $x + 5 = 2 + (2x + 2)$, so that $x = 1$.
   (e) $\log_2 x - \log_2(x-1) = 3$. Condense the LHS and write in exponential form to get $\dfrac{x}{x-1} = 2^3$. Solve this to get $x = \dfrac{8}{7}$.
   (f) $\log_3 x - 2 = \log_3 4$. Move the 2 to the right, the $\log_3 4$ to the left, and condense the LHS to get $\log_3 \dfrac{x}{4} = 2$. Write in exponential form and solve the equation to get $x = 36$.

(g) $\log_5(x+2) + \log_5(x+3) = \log_5(1-x)$. Condense the LHS and write in exponential form to get $(x+2)(x+3) = 1-x$. Expand, move everything to the LHS, and solve the resulting quadratic equation to get $x = -1$ and $x = -5$. Notice, however, that $x = -5$ cannot be a solution because when you substitute it in the LHS you get a logarithm of a negative number, which is undefined. The only solution is $x = -1$.

(h) $4 + \log_2(9x) = 2$. Move the 4 to the LHS and write in exponential form to get $9x = 2^{-2}$, which gives $x = \dfrac{1}{36}$.