MTH30

Extra-credit review sheet for the break

Due May 1st in class (first day of classes after the break)

Professor Luis Fernandez

NOTE:

- This extra homework will give **10 extra points in the final grade (!).**
- To receive the points you must complete all the exercises and **show all your work**.
- Questions **without work** shown will receive **no credit.**
- **No late submissions accepted.**
- **There are many exercises. Start early in the break.**

1. Sketch the graphs of the following linear equations:

(a) $2x - 3y = 6$ (b) $x + 4y = 8$

2. Find the slope of the lines described by the following information:

(a) With equation $2x - 3y = 8$

(b) Passing through the points $(4, -2)$ and $(5, 1)$

(c) Perpendicular to the line with equation $x - 4y = 1$

3. Write an equation of the line described by the following information:

(a) With slope $-\frac{1}{2}$ and passing through the point $(3, -2)$

(b) Passing through the points $(2, -1)$ and $(-4, -3)$

(c) perpendicular to the line with equation $y = 3x - 4$ and passing through $(1, 9)$. the same y–intercept as the line with equation $x - 4y - 8 = 0$.

4. For each of the the following quadratic functions $f(x)$:

A. $f(x) = (x - 2)^2 - 1$ B. $f(x) = x^2 + 2x - 3$

(a) Find the vertex.

(b) Find the x-intercept(s).

(c) Find the y-intercept(s).

(d) Sketch the graph of $y = f(x)$.

5. The graph of a parabola $y = f(x)$ has axis of symmetry $x = -1$, vertex $(-1, 5)$, and $f(0) = 3$.

(a) Write the equation of the parabola in standard form.

(b) Sketch a graph of $y = f(x)$.

6. For each of the the following polynomials $p(x)$:

A. $p(x) = x^3 - 3x^2 + 4$ B. $p(x) = -x^3 + 4x^2 - x - 6$

(a) List all possible rational roots of $p(x)$, according to the Rational Zeros Theorem.

(b) Factor $p(x)$ completely.

(c) Find all roots of the equation $p(x) = 0$.

(d) Determine the end behavior of the graph of $y = p(x)$.

(e) Determine the y–intercept of the graph of $y = p(x)$

(f) Determine the x–intercepts of the graph $y = p(x)$

(g) Determine the local behavior of $y = p(x)$ near the x-intercepts.

(h) Use the above information to sketch a graph of $y = p(x)$.

7. (a) State carefully the remainder theorem.

(b) Find the remainder of the division of $x^{122} - 20x^{51} + 60x^{34} + 1$ when divided by $x - 1$.

(c) State carefully the factor theorem.

(d) Find a polynomial of degree 4 with zeros at $x = 2$ and $x = 1$.

8. For each of the following rational functions f

A. $f(x) = \dfrac{x+1}{x-2}$ B. $f(x) = \dfrac{x^2-9}{x^2-x-2}$ C. $f(x) = \dfrac{x^2}{x^2+1}$

(a) Factor numerator and denominator and simplify if possible.

(b) Find the x and y intercepts of the graph of $y = f(x)$ if they exist.

(c) Find any vertical or horizontal asymptotes.

(d) Use the above information to sketch a graph of $y = f(x)$.

9. For the following functions, find $f(-3)$, $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, $f(2)$, $f(3)$. Then plot the points you got and sketch their graph.

A. $f(x) = 3^x$ B. $f(x) = (\frac{1}{2})^x$ C. $f(x) = 2^{-x}$

10. Convert the following from exponential form to logarithmic form.

A. $e^x = 5$. B. $4^{x+3} = 7$ C. $\left(\dfrac{1}{3}\right)^{2y+1} = x - 3$ D. $10^{x+2} = 14$.

11. Convert the following from exponential form to logarithmic form.

A. $\operatorname{Ln} y = 7$. B. $\log_5(y+3) = x + 7$ C. $\log_{\frac{1}{3}}(2y+1) = 5$ D. $\log(x+2) = 12$.

12. Expand

(a) $\log_7 \left(x^4 y^3\right)$

(b) $\log_3 \dfrac{x^4 y^3}{z^2 w^8}$

(c) $\log \left(x^4 y^3\right)^5$

(d) $\log \sqrt[4]{\dfrac{10x^2y^3}{5z}}$

13. Condense

(a) $3 \log x + 7 \log y$

(b) $4 \log_4 x - 5 \log_4 y + \log_4 z - 3 \log_4 w$

(c) $\dfrac{1}{2} \operatorname{Ln} x - \dfrac{2}{6} \operatorname{Ln} y + \dfrac{3}{4} \operatorname{Ln} z$

(d) $\dfrac{1}{5}(2 \log x - \frac{1}{2} \log y + \frac{2}{3} \log z)$

14. Evaluate the following expressions. Give exact values whenever possible:

(a) $\log_2 \dfrac{1}{64}$

(b) $\log_9 \dfrac{\sqrt{3}}{3}$

(c) $\log_b x^3 y$, given that $\log_b x = 2$ and $\log_b y = 36$

(d) e^{x-y} given that $e^x = 3$ and $e^y = 4$

(e) $\log_a \left(\dfrac{x}{y}\right)$ given that $\log_a(x) = 12$ and $\log_a(y) = 4$

(f) $\ln e^{\sqrt{2}}$

(g) $\log 1000$

(h) $\log_7 31$, rounded to the nearest hundredth

(i) $e^{\text{Ln}\,5}$

(j) $\log_7 7^{124}$

15. Write the following logarithms in the indicated base. Simplify what you can.

(a) $\log_5 7$, in base 7.

(b) $\log_8 4$, in base 2.

(c) $\log_6 10$, in base e.

16. Solve the following equations. If the answer is not an exact numbers, leave it expressed as a logarithm.

(a) $7^{x+2} = 49$

(b) $e^x = 2$

(c) $3^{x+5} = 9 \cdot 3^{x+2}$

(d) $\log_2 x - \log_2(x-1) = 1$

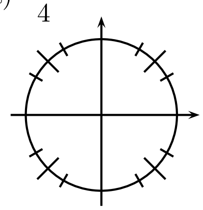
(e) $\log_3 x - 2 = \log_3 4$

(f) $\log_5(x+2) + \log_5(x+3) = \log_5(1-x)$

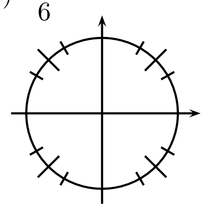
(g) $4 + \log_2(9x) = 2$

17. Draw the following angles in standard position in the circles provided.

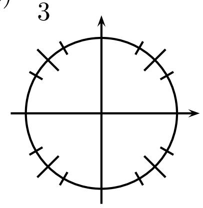
(a) $\frac{3\pi}{4}$



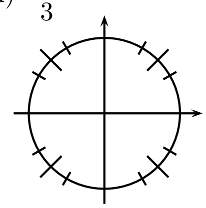
(b) $\frac{7\pi}{6}$



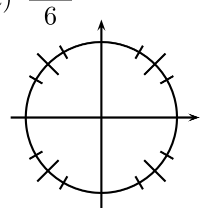
(c) $\frac{2\pi}{3}$



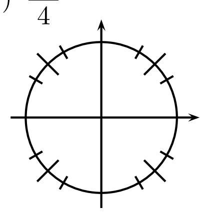
(d) $\frac{5\pi}{3}$



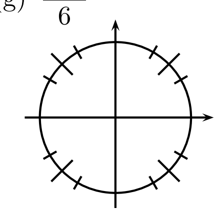
(e) $\frac{11\pi}{6}$



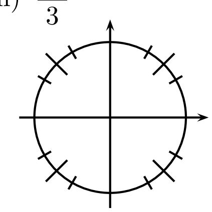
(f) $\frac{7\pi}{4}$



(g) $\frac{17\pi}{6}$

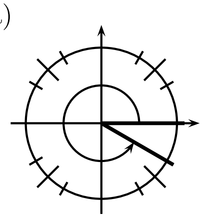


(h) $\frac{7\pi}{3}$

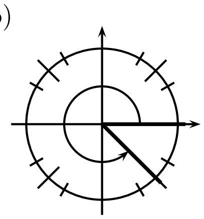


18. Write, in the space provided, the value IN RADIANS of the angles given in the following pictures.

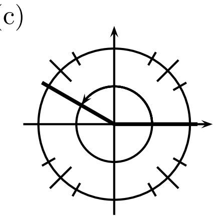
(a)



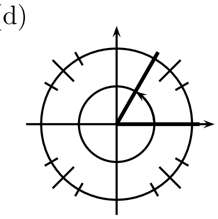
(b)



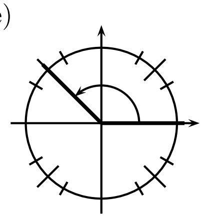
(c)



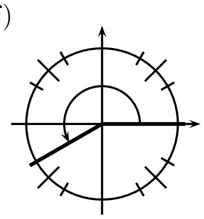
(d)



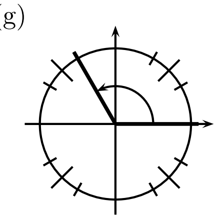
(e)



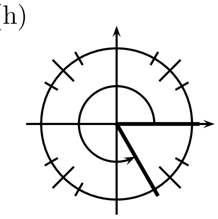
(f)



(g)



(h)



19. Find an angle (in degrees) between $0°$ and $360°$ that is coterminal with the following angles:

(a) $425°$ (b) $1225°$ (c) $-560°$

20. Find an angle (in radians) between 0 and 2π that is coterminal with the following angles:

(a) 11π (b) $\dfrac{11\pi}{2}$ (c) $\dfrac{19\pi}{4}$ (d) $\dfrac{-9\pi}{2}$

21. Find the reference angle of the following angles (in degrees).

(a) $115°$ (b) $267°$ (c) $333°$ (d) $-100°$

22. Find the reference angle of the following angles (in radians).

(a) $\dfrac{7\pi}{6}$ $\dfrac{5\pi}{4}$ $\dfrac{2\pi}{3}$ $-\dfrac{5\pi}{6}$

23. Convert from radians to degrees.

(a) $\dfrac{7\pi}{6}$ (b) $\dfrac{5\pi}{4}$ (c) $\dfrac{5\pi}{3}$ (d) $-\dfrac{11\pi}{6}$

24. Convert from degrees to radians.

(a) $150°$ (b) $240°$ (c) $315°$ (d) $-150°$

25. Given that $\cos\alpha = -\dfrac{4}{5}$, and that α is in Quadrant II, find the exact value of $\sin\alpha$, $\tan\alpha$, $\sec\alpha$, $\csc\alpha$, $\cot\alpha$.

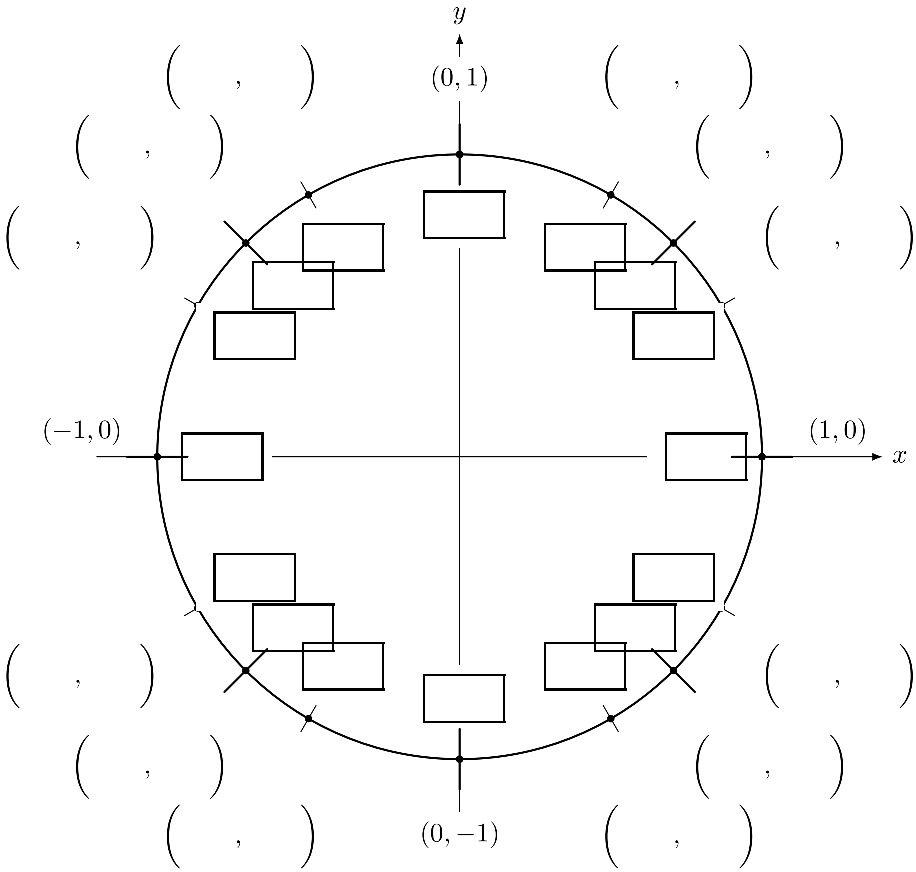
26. Given that $\tan\alpha = -\dfrac{2}{3}$, and that α is in Quadrant IV, find the exact value of $\sin\alpha$, $\cos\alpha$, $\sec\alpha$, $\csc\alpha$, $\cot\alpha$.

27. For the following sinusoidal functions, find the amplitude, the period, and the phase shift.
For (a) and (c), graph a full period of the function.

(a) $f(x) = 2\sin\left(3x - \dfrac{3\pi}{2}\right)$ (b) $f(x) = -6\sin\left(4x - \dfrac{\pi}{2}\right)$

(c) $f(x) = \dfrac{4}{3}\cos\left(2\pi x - \dfrac{\pi}{2}\right)$ (d) $f(x) = 4\sin\left(3x + \dfrac{3\pi}{4}\right)$

28. Fill in the angles, **in radians**, inside the boxes. Then fill in the coordinates of the points marked in the circle. And remember that the sine of an angle is the y coordinate, and the cosine is the x coordinate.



29. Prove the following trigonometric identities.

(a) $\sec^2 x(1-\cos^2 x) = \tan^2 x$

(b) $\sin x\,(\cot x + \csc x) = \cos x + 1$

(c) $\cos^2 x(1+\tan^2 x) = 1$

(d) $\sin x\ \tan x = \sec x - \cos x$.

(e) $\sec x - \cos x = \tan x\ \sin x$.

(f) $\sec x\ \csc x = \tan x + \cot x$.

30. Solve the following equations, for x in the interval $0 \le x < 2\pi$.

(a) $4\sin x - 1 = 2\sin x$

(b) $7\cos x + 1 = 5\cos x$

(c) $3\sin x + 1 = \sin x$

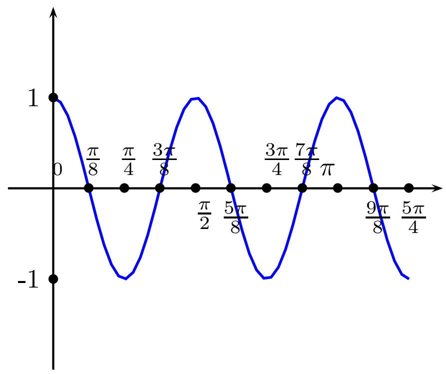
(d) $4\sin x + \sqrt{3} = 2\sin x$

(e) $\cos x - 1 = -\cos x$

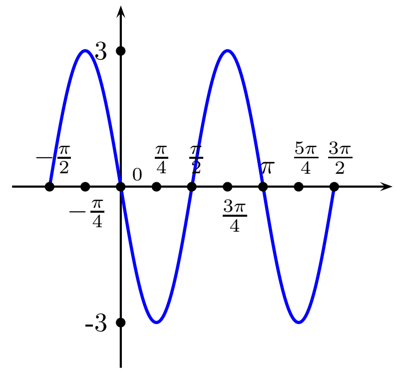
(f) $(\tan x)^2 = 1$

31. The following are the graphs of functions of the form $f(x) = A\sin(Bx - C)$, with $A > 0$. Find the amplitude, the period, and the phase shift. Then use this information to find the values of A, B and C (recall that A will be equal to the amplitude, that the period is $2\pi/B$, and that the phase shift equals B/C).

(a)



(b)



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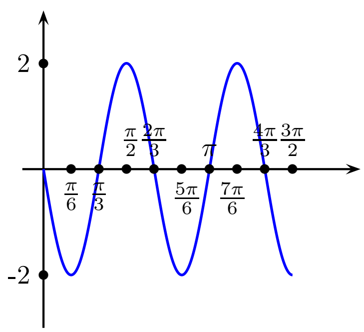

(d)