

### 1.3. Fractions. Professor Luis Fernández

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#### Equivalent fractions

Recall: two fractions  $\frac{a}{b}$  and  $\frac{c}{d}$  are equivalent if they represent the same number, and then we write  $\frac{a}{b} = \frac{c}{d}$ .

In other words, two fractions  $\frac{a}{b} = \frac{c}{d}$  are equivalent if we can multiply and/or divide  $a$  and  $b$ , BOTH BY THE SAME NUMBER, to obtain  $c$  and  $d$ , respectively.

For example:  $\frac{8}{12} = \frac{2}{3}$  because if we divide 8 and 12 by the same number (4) we get 2 and 3, respectively:

$$8 \div 4 = 2 \text{ and } 12 \div 4 = 3.$$

For example:  $\frac{9}{12} = \frac{6}{8}$  because if we divide both 9 and 12 by 3, and then we multiply both by 2, we get 6 and 8, respectively:

$$9 \div 3 \cdot 2 = 6 \text{ and } 12 \div 3 \cdot 2 = 8.$$

Practice exercises: Determine if the given fractions are equivalent.

1.  $\frac{5}{15}$  and  $\frac{1}{3}$       2.  $\frac{2}{8}$  and  $\frac{2}{4}$       3.  $\frac{3}{4}$  and  $\frac{12}{16}$       4.  $\frac{4}{6}$  and  $\frac{10}{15}$

Write a fraction that is equivalent to the given one (note: there are many valid answers).

5.  $\frac{15}{21} = \frac{5}{7}$  (example)      6.  $\frac{2}{3} = \underline{\hspace{2cm}}$       7.  $\frac{7}{2} = \underline{\hspace{2cm}}$   
8.  $\frac{18}{12} = \underline{\hspace{2cm}}$       9.  $\frac{14}{7} = \underline{\hspace{2cm}}$       10.  $\frac{4}{5} = \underline{\hspace{2cm}}$

Recall: simplifying a fraction means to find an equivalent fraction whose numerator and denominator are smaller than the original.

To simplify a fraction, divide BOTH its numerator and denominator by the SAME number, if possible. Keep doing this until there are no common factors of the numerator and the denominator.

When one cannot simplify any further, we say that the fraction is *in lowest terms*.

Practice exercises: Write the given fraction in lowest terms.

11.  $\frac{20}{12} = \underline{\hspace{2cm}}$       12.  $\frac{8}{6} = \underline{\hspace{2cm}}$       13.  $\frac{15}{10} = \underline{\hspace{2cm}}$   
14.  $\frac{2}{8} = \underline{\hspace{2cm}}$       15.  $\frac{15}{6} = \underline{\hspace{2cm}}$       16.  $\frac{24}{12} = \underline{\hspace{2cm}}$

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#### Multiplication of fractions

Remember:  $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ . For example:  $\frac{5}{3} \cdot \frac{2}{7} = \frac{5 \cdot 2}{3 \cdot 7} = \frac{10}{21}$ .

Do not forget to simplify the final function and leave it in lowest terms.

For example:  $\frac{2}{5} \cdot \frac{10}{3} = \frac{2 \cdot 10}{5 \cdot 3} = \frac{20}{15} = \frac{4}{3}$ .

Practice exercises: Multiply and give the final answer in lowest terms.

17.  $\frac{3}{7} \cdot \frac{5}{3} = \frac{3 \cdot 5}{7 \cdot 3} = \frac{15}{21} = \frac{5}{7}$  (ex.)      18.  $\frac{3}{7} \cdot \frac{3}{2} = \underline{\hspace{2cm}}$       19.  $\frac{5}{4} \cdot \frac{7}{3} = \underline{\hspace{2cm}}$   
20.  $\frac{1}{4} \cdot \frac{3}{4} = \underline{\hspace{2cm}}$       21.  $\frac{3}{1} \cdot \frac{2}{7} = \underline{\hspace{2cm}}$       22.  $\frac{6}{8} \cdot \frac{4}{9} = \underline{\hspace{2cm}}$   
23.  $\frac{7}{3} \cdot \frac{3}{8} = \underline{\hspace{2cm}}$       24.  $\frac{1}{5} \cdot \frac{4}{6} = \underline{\hspace{2cm}}$       25.  $\frac{2}{5} \cdot \frac{5}{1} = \underline{\hspace{2cm}}$

## Division of fractions

Remember: To divide two fractions, first rewrite it, changing the division sign to a multiplication sign, and flipping the second fraction. Then multiply and simplify as you did before.

Example:  $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \cdot \frac{7}{5} = \frac{21}{20}$ .

Example:  $\frac{1}{4} \div 3 = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$ .

Example:  $\frac{10}{9} \div \frac{5}{6} = \frac{10}{9} \cdot \frac{6}{5} = \frac{10 \cdot 6}{9 \cdot 5} = \frac{2 \cdot 2}{3 \cdot 1} = \frac{4}{3}$ .

Divide the following fractions. Note that you **always** need to rewrite it first using multiplication instead of division, as explained above.

26.  $\frac{3}{7} \div \frac{5}{2} =$  (ex.)

27.  $\frac{6}{5} \div \frac{7}{9} =$

28.  $\frac{5}{6} \div \frac{2}{9} =$

29.  $4 \div \frac{4}{7} =$

30.  $7 \div \frac{6}{4} =$

31.  $\frac{5}{6} \div 5 =$

32.  $\frac{3}{4} \div 7 =$

33.  $\frac{8}{5} \div 4 =$

34.  $\frac{1}{3} \div \frac{1}{7} =$

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## Addition of fractions

### Fractions with the same denominator

Recall: To add two fractions with the same denominator, just add the numerators and leave the same (common) denominator; then simplify the result as before.

Practice exercises: Add the following fractions.

35.  $\frac{3}{7} + \frac{4}{7} = \frac{7}{7} = 1$  (example)

36.  $\frac{4}{12} + \frac{5}{12} =$

37.  $\frac{6}{5} + \frac{7}{5} =$

38.  $\frac{3}{12} + \frac{14}{12} =$

39.  $\frac{4}{9} + \frac{2}{9} =$

40.  $\frac{1}{15} + \frac{42}{15} =$

### Fractions with different denominators

Recall: To add fractions with different denominators, first find equivalent fractions that have **the same** denominator, and then add them as above.

For example,  $\frac{2}{5} + \frac{4}{3} = \frac{6}{15} + \frac{20}{15} = \frac{26}{15}$ .

How do we find the common denominator? The point is to find a number that can be divided by the given denominators. **Any** number is good.

One option that always works is to use the product of the denominators as the new denominator, as in the last example. But sometimes the new denominator is a very big number, and it is easier to use the Least Common Multiple of the denominators as the common denominator.

For example,  $\frac{7}{15} + \frac{5}{20} = \frac{28}{60} + \frac{15}{60} = \frac{43}{60}$ .

Add the following fractions. Make sure to write the final answer in lowest terms.

41.  $\frac{4}{5} + \frac{3}{4} = \frac{16}{20} + \frac{15}{20} = \frac{31}{20}$  (ex.)

42.  $\frac{4}{3} + \frac{2}{6} = \text{---} + \text{---} =$

43.  $\frac{2}{5} + \frac{7}{4} = \text{---} + \text{---} =$

44.  $\frac{5}{4} + \frac{1}{6} = \text{---} + \text{---} =$

45.  $\frac{5}{12} + \frac{3}{4} = \text{---} + \text{---} =$

46.  $\frac{3}{7} + \frac{3}{4} = \text{---} + \text{---} =$

47.  $\frac{5}{8} + \frac{2}{5} = \text{---} + \text{---} =$

48.  $\frac{5}{6} + \frac{1}{4} = \text{---} + \text{---} =$

49.  $\frac{7}{3} + \frac{6}{5} = \text{---} + \text{---} =$

50.  $\frac{3}{7} + \frac{1}{2} = \text{---} + \text{---} =$

51.  $\frac{2}{6} + \frac{3}{5} = \text{---} + \text{---} =$

52.  $\frac{8}{10} + \frac{7}{4} = \text{---} + \text{---} =$

53.  $\frac{1}{2} + \frac{5}{6} =$

54.  $\frac{1}{1} + \frac{2}{3} =$

55.  $1 + \frac{3}{4} =$

56.  $\frac{5}{3} + \frac{7}{4} =$

57.  $2 + \frac{2}{5} =$

58.  $4 + \frac{3}{2} =$

**Subtraction of fractions**

Subtraction is done using the same idea, only that in the end you subtract the numerators instead of subtracting.

For example:  $\frac{3}{5} - \frac{1}{4} = \frac{12}{20} - \frac{5}{20} = \frac{7}{20}$ .

Subtract the following fractions:

59.  $\frac{5}{4} - \frac{1}{6} = \text{---} - \text{---} =$

60.  $\frac{5}{12} - \frac{1}{4} = \text{---} - \text{---} =$

61.  $\frac{4}{7} - \frac{2}{5} = \text{---} - \text{---} =$

62.  $\frac{5}{8} - \frac{2}{5} = \text{---} - \text{---} =$

63.  $\frac{5}{6} - \frac{1}{4} = \text{---} - \text{---} =$

64.  $\frac{7}{3} - \frac{6}{5} = \text{---} - \text{---} =$

65.  $\frac{5}{4} - \frac{2}{3} =$

66.  $\frac{5}{3} - \frac{1}{9} =$

67.  $\frac{1}{1} - \frac{2}{5} =$

68.  $\frac{1}{1} - \frac{1}{6} =$

69.  $1 - \frac{1}{4} =$

70.  $\frac{8}{7} - \frac{1}{1} =$

71.  $\frac{5}{2} - 1 =$

72.  $\frac{9}{3} - \frac{4}{7} =$

73.  $\frac{7}{2} - \frac{3}{5} =$