## 8.1. Simplify expressions with roots Professor Luis Fernández

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**Square roots.**

Recall: Remember the definition of *square root*:

- **Square:** If $x^2 = y$, then $y$ is the **square** of $x$.
- **Square root:** If $x^2 = y$, then $x$ is the **square root** of $y$.

For example:

- 4 is the square of 2.   2 is the *square root* of 4.
- 81 is the square of 9.   9 is the *square root* of 81.

Exercises

**1.** Write down the squares of all the whole numbers from 0 to 13.

**2.** Write down the **square root** of 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, and 169.

**3.** In your calculator, find the square root sign and find the square root of 225 and 625. Then find the square root of 2, 3, 6, 8, and 10.

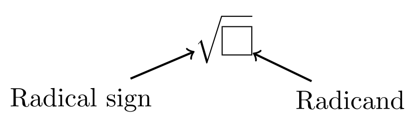
You can see from the last exercise that the square root of most numbers is not a whole number. This is why we have the following notation:

Recall: The notation for the square root is the *radical sign* is "$\sqrt{\phantom{x}}$":

- $\sqrt{x}$ is read *the square root of $x$*.
- So, if $x^2 = y$, then $x = \sqrt{y}$.

In other words, "$\sqrt{\square}$" is the opposite of "$\square^2$", the same way as subtraction is the opposite of addition, or division is the opposite of multiplication.

In the expression "$\sqrt{\square}$", $\sqrt{\phantom{x}}$ is called the *radical sign*, and $\square$ is called the *radicand*:




Exercise:

**4.** Write down $\sqrt{0}$, $\sqrt{1}$, $\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{36}$, $\sqrt{49}$, $\sqrt{64}$, $\sqrt{81}$, $\sqrt{100}$, $\sqrt{121}$, $\sqrt{144}$, and $\sqrt{169}$.

Notice something: the square root of a negative number is not a real number. Why? At the end of this chapter we will study imaginary numbers, which will give meaning to the square root of a negative number.

**Higher order roots.**

Exactly the same as we did with second powers can be done with any power:

- **Cube:** If $x^3 = y$, then $y$ is the **cube** of $x$.
- **Cube root:** If $x^3 = y$, then $x$ is the **cube root** of $y$.

Or in general:

- $\boldsymbol{n}^{\text{th}}$ **power:** If $x^n = y$, then $y$ is the $\boldsymbol{n}^{\text{th}}$ **power** of $x$.
- $\boldsymbol{n}^{\text{th}}$ **power:** If $x^3 = y$, then $x$ is the $\boldsymbol{n}^{\text{th}}$ **root** of $y$.

Thus, we will talk about *fourth* roots, or *sixth* roots, etc.

Exercises

**5.** Write down the cubes of all the whole numbers from 0 to 10. You can use a calculator.

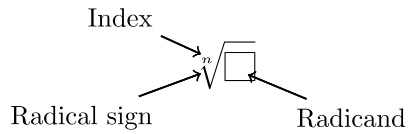
**6.** Write down the **cube root** of 0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

**7.** In your calculator, find how to do the $n^{\text{th}}$ root of any number. Find the cube root of 125 and 64. Then find the cube root of 2, 3, 6, 8, and 10.

**8.** Use your calculator to find the fifth root of 6 and 10.

To express the $n^{\text{th}}$ root we use the symbol "$\sqrt[n]{\square}$".

- The $n$ is called the *index*.
- The "$\sqrt{\phantom{x}}$" is called (as before) *radical sign*.
- The "$\square$" is called (as before) the *radicand*:




Exercises

**9.** Write down $\sqrt[3]{0}$, $\sqrt[3]{1}$, $\sqrt[3]{8}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$, $\sqrt[3]{125}$, $\sqrt[3]{216}$, $\sqrt[3]{343}$, $\sqrt[3]{512}$, $\sqrt[3]{729}$, $\sqrt[3]{1000}$.

**10.** Use your calculator to find $\sqrt[5]{12}$, $\sqrt[8]{4}$, and $\sqrt[12]{17}$.

As before, notice that if $n$ is an even number, then no matter what $x$ is, $x^n$ will be positive. Hence,

if $n$ is even, then the $n^{\text{th}}$ root of a negative number is *not* a real number.

For example, $\sqrt[4]{16} = 2$, but $\sqrt[4]{-16}$ is *not* a real number (it is an imaginary number which we will learn about at the end of this chapter).

### Estimating roots

As you saw in the calculator exercises above, most roots do not give a whole number. However, we can get an idea of how much they are. For example, $\sqrt{12}$ is some number between 3 and 4, because $3^2 = 9$ and $4^2 = 16$, and 12 is between 9 and 16.

Another example: $\sqrt[3]{75}$ is some number between 4 and 5, because $4^3 = 64$ and $5^3 = 125$, andn 75 is between 64 and 125.

Exercise

**11.** Estimate the following roots: $\sqrt{30}$, $\sqrt{50}$, $\sqrt{93}$, $\sqrt[3]{36}$, $\sqrt[5]{50}$.

**12.** Use your calculator to find $\sqrt{30}$, $\sqrt{50}$, $\sqrt{93}$, $\sqrt[3]{36}$, $\sqrt[5]{50}$ and compare with the results in the previous exercise.

### Simplifying expressions with roots

Note: $\sqrt{5^2} = 5$, because $5^2 = 5^2$.

In general,

$\sqrt[n]{x^n} = x$ when $x$ is positive (just because $x^n = x^n$!)

Exercise

**13.** Find $\sqrt{4^2}$, $\sqrt{9^2}$, $\sqrt{234567^2}$, $\sqrt[3]{7^3}$, $\sqrt[7]{3^7}$, $\sqrt[25]{234^{25}}$, $\sqrt[5]{123456^5}$.

When $x$ is negative notice that, when $n$ is even, $x^n$ will be positive, and $\sqrt[n]{x^n}$ will also be positive. Thus, what we wrote above does not work when $x$ is negative. However, one just gets the absolute value of the number:

- If the index $n$ is odd, then $\sqrt[n]{x^n} = x$ always.
- If the index $n$ is even, then $\sqrt[n]{x^n} = |x|$. That is,
  - $\rightarrow$ If $x$ is positive, $\sqrt[n]{x^n} = x$.
  - $\rightarrow$ If $x$ is negative, $\sqrt[n]{x^n} = -x$.

Examples: $\sqrt{x^2} = |x|$. $\sqrt{(-7)^2} = 7$. $\sqrt[5]{x^5} = x$. $\sqrt[4]{x^4} = |x|$. $\sqrt[7]{(-6)^7} = (-6)$.

Exercise:

**14.** Simplify $\sqrt{x^2}$, $\sqrt{t^2}$, $\sqrt{z^2}$, $\sqrt[3]{(-4)^3}$, $\sqrt[6]{(-5)^6}$, $\sqrt[25]{x^{25}}$, $\sqrt[5]{x^5}$.

This idea can be used to simplify roots in general. Recall the *rule of exponents*

$$x^{m \cdot n} = (x^m)^n.$$

Thus, if we can write the radicand as a multiple of the index, then we can simplify the root easily.

Example: Simplify $\sqrt{5^{12}}$.

Since we can write the index, which is 12, as $2 \cdot 6$, we can rewrite $5^{12}$ as $(5^6)^2$. Thus we have

$$\sqrt{5^{12}} = \sqrt{(5^6)^2} = 5^6.$$

Example: Simplify $\sqrt[3]{27y^6}$.

Since we can write $27y^6$ as $(3y^2)^3$, we have $\sqrt[3]{27y^6} = \sqrt[3]{(3y^2)^3} = 3y^2$.

Example: Simplify $\sqrt[4]{16x^{12}}$.

Since we can write $16x^{12}$ as $(2x^3)^4$, we have $\sqrt[4]{16x^{12}} = \sqrt[4]{(2x^3)^4} = 2|x|^3$. (Remember that when the index is even, you need to write the absolute value.

Exercise:

**15.** Simplify $\sqrt[3]{x^9}$

**16.** Simplify $\sqrt[7]{x^{21}}$

**17.** Simplify $\sqrt{x^4}$

**18.** Simplify $\sqrt[4]{x^{32}}$

**19.** Simplify $\sqrt[9]{z^{36}}$

**20.** Simplify $\sqrt[11]{z^{22}}$

**21.** Simplify $\sqrt[4]{81x^{12}}$

**22.** Simplify $\sqrt[7]{128y^{35}}$

**23.** Simplify $\sqrt[3]{125z^{35}}$

**24.** Simplify $\sqrt{9x^{32}y^{18}}$

**25.** Simplify $\sqrt{36x^2y^4z^6}$

**26.** Simplify $\sqrt{-9}$

**27.** Simplify $\sqrt{-16}$

**28.** Simplify $\sqrt[6]{-64}$

**29.** Simplify $\sqrt[3]{-8}$