6.3. Factor special products Professor Luis Fernández

Factoring perfect square trinomials

Recall: we saw in section 5.3 that

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

These polynomials are called *perfect squares* (because they equal $(a+b)^2$). If we are given a trinomial, such as x^2+6x+9, how do we know if it is a perfect square? One only has to check that:

- Two of the terms are squares (say a^2 and b^2).
- The other term is plus or minus 2 times a times b.
- If it is a perfect square, then it factors as $(a+b)^2$ if the middle term has a positive sign and as $(a-b)^2$ if the middle term has a negative sign.

Example: Factor x^2+6x+9. We see that the first term (x^2) is a square, and the last term (9) is also a square (it is 3^2). On the other hand, the other term is $6x = 2\cdot 3\cdot x$. Therefore x^2+6x+9 is a perfect square and $x^2+6x+9=(x+3)^2$.

Note that these polynomials can also be factored using the methods of the previous chapter: we would only need to find two numbers m and n such that $m\cdot n = 9$ and $m+n=6$. It is easy to see that 3 and 3 works, so $x^2+6x+9=(x+3)(x+3)=(x+3)^2$.

Example: Factor $4x^2-12xy+9y^2$. We see that the first term ($4x^2=(2x)^2$) is a square, and the last term ($9y^2=(3y)^2$) is also a square. On the other hand, the middle term is $12xy = 2\cdot(2x)\cdot(3y)$. Therefore $4x^2-12xy+9y^2$ is a perfect square. Since the middle term has a negative sign, we get $4x^2-12xy+9y^2=(2x-3y)^2$.

Practice exercises: Factor the following trinomials.

1. $x^2-6x+9=$ **2.** $x^2+12x+36=$ **3.** $9x^2+6x+1=$

4. $9x^2-12x+4y^2=$ **5.** $9+30x+25x^2=$ **6.** $64x^2+112xy+49y^2=$

Factoring binomials: differences of squares

So far we have learned to factor trinomials (using the methods of the previous sections) and polynomials with 4 terms (using factoring by grouping). What about binomials?

Besides common factors, a binomial can only be factored if it is a difference of squares. Recall:

$$(a+b)(a-b)=a^2-b^2.$$

Reading it backwards gives the key to factor differences of squares:

$$a^2-b^2=(a+b)(a-b).$$

Thus, if we see a polynomial that has the form a^2-b^2, we can always factor it as $(a+b)(a-b)$.

Example: Factor x^2-9. We see that the first term (x^2) is a square, and the last term (9) is also a square (it is 3^2). Since it is a difference of squares (it is x^2-3^2), it factors as $(x+3)(x-3)$.

Example: Factor x^4-4y^2. We see that the first term ($x^4=(x^2)^2$) is a square, and the last term ($4y^2=(2y)^2$) is also a square. Since it is a difference of squares (it is $(x^2)^2-(2y)^2$, it factors as $(x^2+2y)(x^2-2y)$.

Practice exercises: Factor the following binomials.

7. $x^2-9=$ **8.** $x^2-36=$ **9.** $9x^2-1=$

10. $9x^2-4y^2=$ **11.** $9-25x^2=$ **12.** $64x^2-49y^2=$