

### 5.3. Multiplying polynomials Professor Luis Fernández

#### Multiplying a monomial by a polynomial

Recall the distributive property:

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac.$$

If we want to multiply a monomial, such as  $3x^2$ , by a polynomial, such as  $2x^3 - 3x + 5$ , we just distribute and multiply the monomials as we did in the previous section:

$$x^2(2x^3 - 3x + 5) = 2x^5 - 3x^3 + 5x^2$$

Example: Multiply  $7x^2y(3xy - 5 + y^3)$

$$7x^2y(3xy - 5 + y^3) = 7x^2y(3xy - 5 + y^3) = 21x^3y^2 - 35x^2y + 7x^2y^4$$

Practice exercises: Multiply

- |                                |                                  |
|--------------------------------|----------------------------------|
| 1. $2x(x^3 - 7x^2 + 4x + 6) =$ | 2. $-4x^2(-2x^2 - 5x + 6) =$     |
| 3. $2xy(x - 2y + xy) =$        | 4. $(3 - 3x^2 + 6x) \cdot x^2 =$ |
| 5. $-4xy^2(-2x^2y - 5) =$      | 6. $-2x(x - 2) =$                |

#### Multiplying two polynomials

To multiply two polynomials in general you just have to distribute twice. In the end, it amounts to multiplying every term in the first polynomial with every term in the second, and add the results. For example,

$$(a + b)(c + d + e) = (a + b)(c + d + e) = ac + ad + ae + bc + bd + be.$$

It is important to keep organized when you are multiplying polynomials. I recommend that you do it as follows:

- 1) Multiply the first term in the first polynomial with each and all of the terms in the second polynomial and write the results. Take a break.
- 2) Multiply the second term in the first polynomial with each and all of the terms in the second polynomial and write it down. Take a break.
- 3) Continue until you have used all the terms in the first polynomial.
- 4) Combine like terms and you are done.

Example: Multiply  $(x + 3)(2x - 1)$ .

$$(x + 3)(2x - 1) = 2x^2 - x + 6x - 3 = 2x^2 + 5x - 3.$$

Example: Multiply  $(x - 4)(x^2 - 3x + 5)$ .

$$(x - 4)(x^2 - 3x + 5) = 4x^3 - 3x^2 + 5x - 4x^2 + 12x - 20 = 4x^3 - 7x^2 + 17x - 20.$$

Practice exercises: Multiply.

- |                         |                           |
|-------------------------|---------------------------|
| 7. $(x + 3)(x + 2) =$   | 8. $(x + 5)(x - 3) =$     |
| 9. $(2x - 3)(5x - 6) =$ | 10. $(x^2 - 5)(-x - 3) =$ |

**11.**  $(x + 3)(x^2 + 3x + 1) =$

**12.**  $(x - 3)(x^2 - 6x - 4) =$

**13.**  $(x^2 + x + 3)(x^2 - 7) =$

**14.**  $(x^2 - 6x - 4)(x - 5) =$

**15.**  $(x^2 - 2x - 3)(x^2 - x - 4) =$

**16.**  $(-3x^2 - 2x + 1)(-x^3 - 5x + 6) =$

**17.**  $x(x + 3)(x + 2) =$

**18.**  $(x + 1)(x + 2)(x + 3) =$

**19.**  $(x + 3)^2 =$

**20.**  $(x + 1)^2 =$

**21.**  $(x - 3)^2 =$

**22.**  $(x - 1)^2 =$

**23.**  $(x + 3)(x - 3) =$

**24.**  $(x - 1)(x + 1) =$

**25.**  $(a + b)^2 =$

**26.**  $(a - b)^2 =$

**27.**  $(a + b)(a - b) =$

### Some special products

Some products are very common and it is necessary for the future to memorize the result. These are the following (memorize them!):

$$(a + b)^2 = a^2 + 2ab + b^2.$$

$$(a - b)^2 = a^2 - 2ab + b^2.$$

$$(a + b)(a - b) = a^2 - b^2.$$

For example,

$$(x - 5)^2 = x^2 - 10x + 25.$$

$$(x + 7)^2 = x^2 + 14x + 49.$$

$$(x + 4)(x - 4) = x^2 - 16.$$

Practice exercises: Multiply.

**28.**  $(x - 4)^2 =$

**29.**  $(x + 4)^2 =$

**30.**  $(x - 9)^2 =$

**31.**  $(x + 9)^2 =$

**32.**  $(x - 7)^2 =$

**33.**  $(x + 7)^2 =$

**34.**  $(x - 4)(x + 4) =$

**35.**  $(x + 4)(x - 4) =$

**36.**  $(x - 10)(x + 10) =$

**37.**  $(x + 1)(x - 1) =$