2.3. Solving a formula for a specific variable. Professor Luis Fernández

Formulas as equations with more than one variable

Recall: Some equations contain more than one variable. These equations are often called "formulas" because they give some interesting quantity in terms of others. For example, the formula

$$A = \frac{b \cdot h}{2}$$

gives the area A of a triangle with height h and base of length b. Or the formula

$$F = 32 + \frac{9}{5}C$$

gives the formula to convert from C degrees Celsius to degrees Fahrenheit (F).

Example: To find the area of a triangle with base of length 4in and height 6in, one just has to substitute $b = 4$ and $h = 6$ in the formula above:

$$A = \frac{4 \cdot 6}{2} = 12 \text{ sq in.}$$

However, sometimes we would want to find a formula for a different variable. For example, we would want a formula that gives the height of a triangle given its area and its base. To find such a formula, *we will need to solve for one of the variables in terms of the others.*

The procedure is the same as when the equations have only one variable: just treat the variables as if they were numbers and do the same steps.

Example: Solve for h in the formula for the area of a triangle above.

Start with $A = \frac{b \cdot h}{2}$. Multiply both sides by 2 to get $2A = b \cdot h$, and finally divide both sides by b to get $\frac{2A}{b} = h$.

Practice exercises: Solve the following equations for the indicated variable.

1. Solve for T in the formula $PV = nRT$

2. Solve for y in the formula $2x + y = 7$

3. Solve for C in the formula $F = 32 + \frac{9}{5}C$

4. Solve for x in the formula $-5x + 2y = 4$

5. Solve for b in the formula $a^2 + b^2 = h^2$

6. Solve for r in the formula $A = 2\pi r$

7. Solve for B in the formula $A = \frac{h(B + b)}{2}$

8. Solve for y in the formula $3x - 6y = 3$

There is only point in which one has to think a bit more: sometimes the variable we are solving for appears in more than one term. For example, if we want to solve for the variable q in the formula

$$G = pq + rq + 3,$$

we need to combine the terms pq and rq. This is done exactly as with numbers.

For example, to combine $5q + 4q$ we add the 4 and the 5 and we get $9q$. Likewise, to combine $pq + rq$ we add p and r to get $(p + r) \cdot q$.

Example: Solve for A in the formula $K = AC + BC + AB$.

We first move the term without A to the right (that is, subtract BC from both sides) to get $K - BC = AC + AB$. Then combine AC and AB to get $K - BC = A(C + B)$.

Finally divide both sides by $(C + B)$ to get $A = \frac{K - BC}{C + B}$.

Practice exercises: Solve the following equations for the indicated variable.

9. Solve for T in the formula $V = 3T + 5 - RT$

10. Solve for y in the formula $2y + xy - 4 = 6$

11. Solve for a in the formula $3 = ax + ax^2 + 2$

12. Solve for b in the formula $ax + 2y = 3a$