2.1. Operations with integers. Professor Luis Fernández

Absolute value

Remember: The absolute value of a number is the number without its sign. Another way: it is the distance from the number to 0.

It is denoted by two vertical bars. For example, "$|-5|$" means "absolute value of (-5)".

For example, $|12| = 12$, and $|-7| = 7$.

Practice exercises

1. $|11| =$

2. $|-7| =$

3. $|-12| =$

4. $|-234| =$

5. $|56| =$

6. $|0| =$

Inequalities

Remember:

- "$<$" means "less than". For example, "$5 < 8$" means "5 less than 8".
- "$>$" means "greater than". For example, "$6 > 2$" means "6" greater than "2".

To remember which symbol to use, remember that the large side of the symbol "$<$" or "$>$" corresponds to the larger number, and the small side to the smaller number.

Practice exercises. Fill in the blanks with the appropriate symbol "$<$" or "$>$".

7. $5 \square 7$

8. $8 \square 1$

9. $-5 \square 2$

10. $-3 \square 0$

11. $-6 \square -7$

12. $-23 \square -6$

Opposite of a number

Remember: The opposite of a number is the number with the opposite sign. For example, the opposite of 5 is -5 and the opposite of -9 is 9.

It is denoted by writing a $-$ sign in front. For example, "$-(-5)$" means "the opposite of (-5)", and "$-(-(-7))$" means "the opposite of the opposite of (-7)".

Practice exercises. Find the value of the following.

13. $-(-3) =$

14. $-9 =$

15. $-(-67) =$

16. $-(-(-9)) =$

17. $-(-12) =$

18. $-(-(-(-12))) =$

Addition of signed numbers

Remember: to add two signed numbers,

- If the numbers have the same sign, the numbers are working together: add their absolute values and put the same sign that the numbers have.
- If the numbers have different sign, they are working against each other: subtract their absolute values and put the sign of the one with greater absolute value.

Examples:

$5 + (-6)$. The numbers have different sign, so they are competing. Subtract their absolute values: $6 - 5 = 1$. Since (-6) has greater absolute value than 5, (-6) wins, so the sum will have a negative sign. Therefore, $5 + (-6) = (-1)$.

$(-4) + (-5)$. The numbers have the same sign (negative), so they are working together. They join forces, so we add them: $4 + 5 = 9$. Finally, the sign is the common sign they have, giving $(-4) + (-5) = (-9)$.

Practice exercises. Find each sum.

19. $(-5) + 9 =$

20. $(-6) + (-2) =$

21. $(-5) + 5 =$

22. $(-7) + 12 =$

23. $(-3) + (-5) =$

24. $15 + (-2) =$

25. $17 + (-15) =$

26. $(-47) + 37 =$

27. $23 + (-34) =$

28. $(-19) + 9 =$

29. $43 + (-5) =$

30. $4 + (-16) =$

Subtraction

Remember: **Subtracting** is the same as **adding the opposite**.

To subtract two numbers,

1) **Change** the **subtraction** sign **to** an **addition** sign.
2) **Change the second number to its opposite**.
3) **Add** as we learned before.

For example: $3 - (-4) = 3 + 4 = 7$. Another example: $(-6) - 4 = (-6) + (-4) = (-10)$.

Practice exercises. Do the following subtractions.

31. $(-7) - (-6) =$

32. $3 - 9 =$

33. $(-32) - (-1) =$

34. $12 - (-4) =$

35. $49 - (-6) =$

36. $20 - 18 =$

37. $22 - (-10) =$

38. $(-9) - (-9) =$

39. $(-32) - (-12) =$

40. $30 - 62 =$

41. $(-10) - 10 =$

42. $10 - (-15) =$

Multiplication

Remember: signed numbers are multiplied or divided the same way as whole numbers, but then you have to remember to write the right sign at the end. The rule for signs is:

$$+ \cdot + = + \qquad + \cdot - = -$$
$$- \cdot - = + \qquad - \cdot + = -$$

For example: $(-5) \cdot 6 = (-30)$ (because $5 \cdot 6 = 30$ and $- \cdot + = -$).

Practice exercises. Multiply.

43. $(-6) \cdot (-5) = 30$ (example)

44. $(-5) \cdot (7) = -35$ (example)

45. $(-2) \cdot (-3) =$

46. $2 \cdot (-4) =$

47. $(-9)\cdot(-1) =$

48. $(-5)\cdot 10 =$

49. $(-2)\cdot(-2) =$

50. $2\cdot 8 =$

51. $7\cdot(-3) =$

52. $9\cdot(-2) =$

Division

Remember: signed numbers are divided the same way as whole numbers, but then you have to remember to write the right sign at the end. The rule for signs is the same as for multiplication:

$$+\div+=+ \qquad +\div-=-$$
$$-\div-=+ \qquad -\div+=-$$

For example: $(-10)\div 5 = (-2)$ (because $10\div 5 = 2$ and $-\div+=-$).

It can also written with fraction notation:

$$\frac{+}{+}=+ \qquad \frac{+}{-}=-$$
$$\frac{-}{-}=+ \qquad \frac{-}{+}=-$$

For example: $\dfrac{-6}{-2} = 3$ (because $\dfrac{6}{2} = 3$ and $\dfrac{-}{-} = +$).

Practice exercises. Divide

53. $(-12)\div(-6) = 2$ (example)

54. $(-15)\div 3 = (-5)$ (example)

55. $(-2)\div(-1) =$

56. $12\div(-2) =$

57. $(-9)\div(-3) =$

58. $(-50)\div 10 =$

59. $(-26)\div(-2) =$

60. $8\div 2 =$

61. $17\div(-1) =$

62. $18\div(-9) =$

Order of operations

Remember: When several operations are involved in a mathematical expression, it is understood that they are performed in the following order:

- Exponents and roots.
- Multiplication and division, **left to right** in order of appearance.
- Addition and subtraction, **left to right** in order of appearance.

Also, whenever there are **grouping symbols**, do the operations **inside** first. Grouping symbols are:

- Round parenthesis "( )".
- Square parenthesis "[ ]".
- Braces "{ }".
- Any horizontal line, such as $\sqrt{}$, or a fraction line, above or below an expression.

Practice exercises:

63. $11 - 72\div 9$

64. $18 - 42\div 7$

65. $9\cdot 4 + 3$

66. $12\cdot 5 + 20$

67. $18 - 6 + 7$

68. $13 - 5 - 1 + 9\div 3$

69. $5\cdot 6 - (15 - 6)$

70. $3\cdot 9 - (35 - 1)$

71. $9 + 3 - 12$

72. $17 - 10 - 8$

73. $3 \cdot 4 \cdot 2 \div 4 + 3$

74. $2 \cdot 5 \cdot 10 \div 5 + 3$

75. $(2 \cdot 2)^2$

76. $(5 \cdot 2)^2$

77. $13 + 0 \div 7$

78. $9 + 10 \div 5$

79. $[12 \div (4 \div 2)]^2$

80. $[32 \div (8 \div 2)]^2$

81. $12 + 3 \cdot 2 + (3 + 5 \cdot 2)$

82. $9 + 4 \cdot 5 + (8 + 4 \cdot 4)$

83. $13 + 2(5 - 3)$

84. $16 + 5(9 - 4)$

85. $12 \div 3 \cdot 4$

86. $25 \div 5 \cdot 5$

Evaluating expressions with integers

To evaluate a mathematical expression, simply substitute the value of the variable or variables into the expression and simplify the result.

IMPORTANT: when you substitute variables by a number, always write the number **in parenthesis** when you substitute. This way you will avoid confusions and errors.

For example: Evaluate $\dfrac{x+y}{x^2+4}$ when $x = -1$, $y = 2$:

- Substitute each variable by its given value, written in parenthesis: $\dfrac{(-1)+(2)}{(-1)^2+4}$.
- Simplify the expression, remembering to use order of operations appropriately: $\dfrac{(-1)+(2)}{(-1)^2+4} = \dfrac{1}{1+4} = \dfrac{1}{5}$.

NOTE: It is always useful to write a dot "$\cdot$" at those places where we have to multiply but there is no multiplication symbol (just an empty space). So for example, if you have the expression $3mn$, start by writing it as $3 \cdot m \cdot n$. It makes things more clear.

Practice exercises:

Evaluate the following expressions

87. $a + 6b$ if $a = 4$, $b = 8$

88. $a + 6b$ if $a = -7$, $b = -2$

89. $4xy$ if $x = 4$, $y = -3$

90. $x^2 - y^2 + 3$ if $x = 2$, $y = 3$

91. $x^2 - y^2 + 3$ if $x = -2$, $y = 1$

92. $x^2 - y^2 + 3$ if $x = -1$, $y = -3$