Review for test 3, MTH 06.

1. Simplify the following rational expressions.

(a) $\dfrac{\dfrac{y+3}{y}}{\dfrac{y-2}{3y}}$

(b) $\dfrac{2+\dfrac{1}{x}}{1-\dfrac{2}{x}}$

(c) $\dfrac{\dfrac{2}{x}+\dfrac{1}{x-1}}{\dfrac{2}{x-1}+\dfrac{1}{x+1}}$

(d) $\dfrac{\dfrac{x+2}{x-1}+\dfrac{x-2}{x+1}}{\dfrac{x+2}{x-1}-\dfrac{x-2}{x+1}}$

2. Solve the following equations.

(a) $\dfrac{x}{3}+\dfrac{3}{2}=\dfrac{x}{6}+\dfrac{2x}{3}$

(b) $\dfrac{5}{x-3}=\dfrac{4}{x-2}$

(c) $2+\dfrac{5}{x}=\dfrac{2}{x+9}$

(d) $\dfrac{1}{x-4}+\dfrac{1}{x+4}=\dfrac{12}{x^2-16}$

(e) $\dfrac{5}{x-2}+\dfrac{6}{x+2}=2$

(f) $\dfrac{x}{x-4}=\dfrac{5x}{x^2-x-12}-\dfrac{3}{x+3}$

(g) $\dfrac{2}{x^2-4}-\dfrac{1}{x^2+x-2}=\dfrac{3}{x-3x+2}$

3. Do a table of values and graph the following equations.

(a) $y=3\cdot 2^x$.

(b) $y=\left(\dfrac{1}{3}\right)^x$.

4. Find the exact value of the following expressions.

(a) $\log_3 81$.

(b) $\log_2 32$.

(c) $\log_8 2$.

(d) $\log_{16} 8$.

(e) $\log_3 \dfrac{1}{27}$.

(f) $\log_9 \dfrac{1}{27}$.

(g) $\log_{10} \dfrac{1}{10,000}$.

(h) $\log_{81} \dfrac{1}{27}$.

5. Solve the following equations.

    (a) $5^x = 125$.

    (b) $2^x = \dfrac{1}{16}$.

    (c) $4^{x-1} = 16$.

    (d) $2^{2x-3} = 16$.

    (e) $3^{3x-1} = \dfrac{1}{27}$.

6. Solve the following equations.

    (a) $\log_5 x = 3$.

    (b) $\log_b \dfrac{27}{8} = 3$.

    (c) $x = \log_5 125$.

    (d) $\log_8 x = \dfrac{1}{2}$.

    (e) $\log_b \dfrac{1}{8} = -\dfrac{3}{4}$.
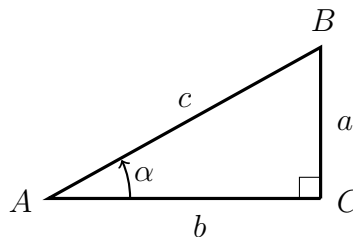
    (f) $\log_{5/7} x = 2$.

    (g) $\log_{2/3} x = -3$.

7. For a triangle as in the figure below, find the exact value of $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\cot\alpha$, $\sec\alpha$, and $\csc\alpha$ given that

    (a) $a = 3$, $b = 4$, $c = 5$.

    (b) $a = 10$, $b = 24$, $c = 26$.

    (c) $a = 5$, $b = 7$, $c = \sqrt{74}$.

    (d) $a = 2$, $b = 5$ (you need to find c also).

For the next 4 exercises you will need a calculator. Round your answers to the nearest hundredth.

8. Solve each right triangle using the given information. Lengths refer to a triangle labeled as the one below. In each case $\angle C = 90°$.

    (a) $\angle A = 35°$, $a = 10$.

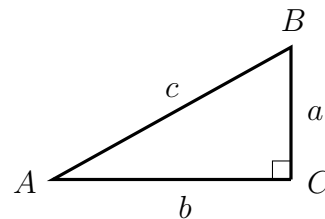   (b) $\angle A = 50°$, $b = 8$.

    (c) $\angle A = 20°$, $c = 12$.

    (d) $\angle B = 20°$, $a = 2$.

    (e) $\angle B = 80°$, $c = 7$.

    (f) $\angle B = 37°$, $b = 4$.

9. The angle of elevation of the top of a tower is $40°$ from an observation point 90 ft. from the base of the tower. Find the height of the tower.

10. Measured from a boat in a lake, the angle of elevation of the top of a tree is $40°$ when the boat is 80 ft from the base of the tree. Find the height of the tree.

11. A 50 ft pole casts a shadow 20 ft long. Find the angle of elevation of the sun.

12. Find the exact value of $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\cot\alpha$, $\sec\alpha$, and $\csc\alpha$ given that:

   (a) $\cos\alpha = \dfrac{3}{5}$ and α is in the first quadrant.

   (b) $\cos\alpha = -\dfrac{2}{3}$ and α is in the second quadrant.

   (c) $\sin\alpha = \dfrac{4}{7}$ and α is in the second quadrant.

   (d) $\sin\alpha = -\dfrac{5}{8}$ and α is in the fourth quadrant.

   (e) $\tan\alpha = \dfrac{4}{3}$ and α is in the third quadrant.

13. Find all the angles α between 0 and 360° that satisfy

   (a) $\sin\alpha = \dfrac{1}{2}$ (write exact values of α).

   (b) $\cos\alpha = -\dfrac{\sqrt{2}}{2}$ (write exact values of α).

   (c) $\sin\alpha = 0.34$ (use a calculator here to find one of the values; find the other using a picture).

   (d) $\cos\alpha = -0.28$ (use a calculator here to find one of the values; find the other using a picture).

14. Fill in the remaining angles inside the boxes. Then fill in the remaining coordinates of the points marked in the circle. [Recall that the x-coordinate of the point is the cosine of the corresponding angle, and the y-coordinate is the sine.]

y

$(\quad , \quad)$

$(0, 1)$

$(\quad , \quad)$

$(\quad , \quad)$

$(\quad , \quad)$

$\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right)$

$(\quad , \quad)$

$135°$

$30°$

$(-1, 0)$

$(1, 0)$

x

$(\quad , \quad)$

$300°$

$(\quad , \quad)$

$\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2} \right)$

$(\quad , \quad)$

$\left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2} \right)$

$(0, -1)$

$(\quad , \quad)$