MTH 05, Test 3, V. 3, 11/20/18 Luis Fernández

NAME: SOLUTION

There are nineteen questions. Multiple choice questions are 5 points each. Free response questions are 7 points each. For multiple-choice questions, circle your answer. For free-response questions, SHOW ALL WORK to receive full credit.

1. Write using only positive exponents:
$$(-x^3y^{-6}z^5)(8x^{-3}yz^4)$$

(a) $\dfrac{z^9}{8y^5}$

(b) $-\dfrac{8z^{20}}{x^9y^6}$

(c) $-\dfrac{8z^9}{y^5}$

(d) $\dfrac{24x^6z^9}{y^5}$

Solution:

$$\begin{aligned}(-x^3y^{-6}z^5)(8x^{-3}yz^4) &= -8x^{3+(-3)}y^{-6+1}z^{4+5}\\ &= -8y^{-5}z^9\\ &= -\frac{8z^9}{y^5}\end{aligned}$$

2. Multiply: $(3x-2)(x^2+4x-5)$

(a) $3x^3-14x^2-23x+10$

(b) $3x^3+10x^2-23x+10$

(c) $12x^6-12x^4+10$

(d) $3x^3+10x^2+7x+10$

Solution:

$$\begin{aligned}&(3x-2)(x^2+4x-5)\\ &= 3x^3+12x^2-15x-2x^2-8x+10\\ &= \boxed{3x^3+10x^2-23x+10}\end{aligned}$$

3. Divide and write in scientific notation:
$$\frac{3.5\times10^7}{5\times10^{-5}}$$

(a) 7×10^{11}

(b) 7×10^{10}

(c) 0.7×10^{12}

(d) 7×10^{12}

Solution:

$$\frac{3.5\times10^7}{5\times10^{-5}} = \frac{3.5}{5}\cdot\frac{10^7}{10^{-5}} = 0.7\times10^{12} = 7\times10^{11}$$

4. Which of the following is a factor of
$$4x^4-100x^2?$$

(a) $x+5$

(b) $4x-10$

(c) x^2+5

(d) 10

Solution:
Factor out the GCF: the GCF of the coefficients is 4. The GCF for x is x^2.
Therefore the GCF is $4x^2$:
$4x^4-100x^2 = 4x^2(x^2-25)$.
The first two factors are monomials, so they cannot be factored further.
The last term (x^2-25) is a difference of squares, which is factored as $(x+5)(x-5)$.
Therefore $4x^4-100x^2 = 4x^2(x+5)(x-5)$.
The only factor that appears as solution is $\boxed{x+5}$.

5. Write with only positive exponents:

$$\left(\frac{12x^2y^{-3}}{4x^{-5}}\right)^{-2}$$

(a) $-9y^6x^{-6}$

(b) $\dfrac{y^6}{9x^{14}}$

(c) $\dfrac{9y^6}{x^9}$

(d) $-\dfrac{6x^6}{y^6}$

Solution:

$$\begin{aligned}\left(\frac{12x^2y^{-3}}{4x^{-5}}\right)^{-2} &= \left(3x^{2-(-5)}y^{-3}\right)^{-2}\\ &= \left(3x^7y^{-3}\right)^{-2}\\ &= 3^{-2}x^{-14}y^6\\ &= \boxed{\frac{y^6}{9x^{14}}}\end{aligned}$$

6. Expand: $(a+b)^2$

(a) a^2-b^2

(b) $a^2+2ab+b^2$

(c) $(a+b)(a-b)$

(d) a^2+b^2

Solution: $(a+b)^2 = \boxed{a^2+2ab+b^2}$, as you should have memorized.

7. Factor: $4x^2-25$.

(a) $2(x-5)(x+5)$

(b) $(2x+5)(2x-5)$

(c) Cannot be factored.

(d) $(2x-5)^2$

Solution:
It has 2 terms, so if it can be factored it is because it is a difference of squares. It is: the first term is $(2x)^2$ and the second is 5^2. Therefore $4x^2-25 = \boxed{(2x+5)(2x-5)}$.

8. Which of the following is a factor of the polynomial $2cx+5cy-6dx-15dy$?

(a) $c+3d$

(b) $2x+5y$

(c) $x-3y$

(d) Cannot be factored

Solution:
It has 4 terms, so factor by grouping:

$$\begin{aligned}&2cx+5cy-6dx-15dy\\ &= c(2x+5y)-3d(2x+5y)\\ &= (2x+5y)(c-3d)\end{aligned}$$

Therefore the answer is $\boxed{\text{2x+5y}}$

9. Factor completely: $4x^2 + 11x - 3$

(a) $(2x+1)(2x-1)$

(b) $(x+3)(4x-1)$

(c) $(x+1)(4x-3)$

(d) Cannot be factored.

Solution:

It is a trinomial that is not monic, so use ac method.
We need two numbers m, n so that
$m + n = 11$
$m \cdot n = -12$
12 and -1 work. Break the middle term $11x$ as $-x + 12x$ and factor by grouping:

$$\begin{aligned} 4x^2 + 11x - 3 &= 4x^2 - x + 12x - 3 \\ &= x(4x-1) + 3(4x-1) \\ &= \boxed{(4x-1)(x+3)} \end{aligned}$$

10. Simplify: $\dfrac{x^4 x^{-7}}{x^5}$.

(a) $\dfrac{1}{x^5}$

(b) x^2

(c) $\dfrac{1}{x^8}$

(d) x^8

Solution:

$$\frac{x^4 x^{-7}}{x^5} = x^{4+(-7)-5} = x^{-8} = \boxed{\frac{1}{x^8}}.$$

11. Simplify. $\dfrac{45x^7 - 27x^3 + 36x^5}{-9x^3}$

(a) $-5x^{21} + 3x^9 - 4x^{15}$

(b) $-5x^4 + 4x^2$

(c) $36x^4 - 36 + 27x^2$

(d) $-5x^4 + 3 - 4x^2$

Solution:

$$\begin{aligned} &\frac{45x^7 - 27x^3 + 36x^5}{-9x^3} \\ &= \frac{45x^7}{-9x^3} + \frac{-27x^3}{-9x^3} + \frac{36x^5}{-9x^3} \\ &= \boxed{-5x^4 + 3 - 4x^2} \end{aligned}$$

12. Simplify $(4x^2 + 5x - 4) - (-6x^2 - 5x + 7)$.

(a) $10x^2 + 10x - 3$

(b) $-2x^2 + 10x + 11$

(c) $10x^2 + 10x - 11$

(d) $-24x^4 - 25x^2 - 28$

Solution:

$$\begin{aligned} &(4x^2 + 5x - 4) - (-6x^2 - 5x + 7) \\ &= (4x^2 + 5x - 4) + (6x^2 + 5x - 7) \\ &= \boxed{10x^2 + 10x - 11} \end{aligned}$$

13. Which of the following is a factor of the polynomial $x^2 - 17x + 30$?

(a) $(x + 2)$

(b) $(x + 15)$

(c) $(x - 17)$

(d) $(x - 15)$

Solution: We want two numbers m and n with
$m + n = -17$,
$m \cdot n = 30$.
These numbers are -2 and -15.
Therefore $x^2 - 17x + 30 = (x - 2)(x - 15)$.
The only factor of these two that appears in the solutions is $\boxed{(x - 15)}$.

14. Give the product in scientific notation.

$$(6 \times 10^3)(7 \times 10^7)$$

(a) 4.2×10^{10}

(b) 4.2×10^{11}

(c) 4.2×10^9

(d) 42×10^{10}

Solution:

$$\begin{aligned}(6 \times 10^3)(7 \times 10^7) &= 42 \times 10^{3+7} \\ &= 42 \times 10^{10} \\ &= 4.2 \times 10^{11}\end{aligned}$$

Free response questions start here. SHOW ALL WORK!!!

15. Factor completely: $3x^3 - 15x^2 + 18x$.

Solution:

Factor out the common factors and then factor the trinomial:

$$\begin{aligned}3x^3 - 15x^2 + 18x &= 3x(x^2 - 5x + 6) \\ &= 3x(x - 2)(x - 3)\end{aligned}$$

16. Divide: $\dfrac{9x^3 - 6x^2}{3x^2}$.

Solution:

$$\begin{aligned}\frac{9x^3 - 6x^2}{3x^2} &= \frac{9x^3}{3x^2} + \frac{-6x^2}{3x^2} \\ &= 3x - 2.\end{aligned}$$

17. Multiply: $(x^2 + 3x - 6)(x - 7)$

Solution:

$$\begin{aligned}(x^2 + 3x - 6)(x - 7) \\ &= x^3 - 7x^2 + 3x^2 - 21x - 6x + 42 \\ &= \boxed{x^3 - 4x^2 - 27x + 42}.\end{aligned}$$

18. Factor completely: $x^6y^3 - 16x^2y^7$

Solution:

Factor the common factors first. Then factor the binomial as a difference of squares. Finally, one of the factors is also a difference of squares, so it can be factored:

$$\begin{aligned}x^6y^3 - 16x^2y^7 \\ &= x^2y^3(x^4 - 16y^4) \\ &= x^2y^3(x^2 + 4y^2)(x^2 - 4y^2) \\ &= x^2y^3(x^2 + 4y^2)(x + 2y)(x - 2y)\end{aligned}$$

19. Multiply: $(7x - 5)(7x + 5)$

Solution:

Use the formula $(a - b)(a + b) = a^2 - b^2$:

$$\begin{aligned}(7x - 5)(7x + 5) &= (7x)^2 - 5^2 \\ &= 49x^2 - 25\end{aligned}$$