# MTH 05, Test 3, V. 2, 11/20/18 Luis Fernández

**NAME:** SOLUTION

**There are nineteen questions. Multiple choice questions are 5 points each. Free response questions are 7 points each. For multiple-choice questions, circle your answer. For free-response questions, SHOW ALL WORK to receive full credit.**

**1.** Write using only positive exponents:
$(-x^3y^{-6}z^5)(8x^{-3}yz^4)$

(a) $-\dfrac{8z^{20}}{x^9y^6}$

(b) $\dfrac{24x^6z^9}{y^5}$

(c) $-\dfrac{8z^9}{y^5}$

(d) $\dfrac{z^9}{8y^5}$

**Solution:**

$$\begin{aligned}(-x^3y^{-6}z^5)(8x^{-3}yz^4) &= -8x^{3+(-3)}y^{-6+1}z^{4+5}\\ &= -8y^{-5}z^9\\ &= -\frac{8z^9}{y^5}\end{aligned}$$

**2.** Which of the following is a factor of the polynomial $x^2 - 17x + 30$?

(a) $(x + 15)$

(b) $(x - 15)$

(c) $(x - 17)$

(d) $(x + 2)$

**Solution:** We want two numbers $m$ and $n$ with
$m + n = -17$,
$m \cdot n = 30$.
These numbers are $-2$ and $-15$.
Therefore $x^2 - 17x + 30 = (x - 2)(x - 15)$.
The only factor of these two that appears in the solutions is $\boxed{(x - 15)}$.

**3.** Simplify $(4x^2 + 5x - 4) - (-6x^2 - 5x + 7)$.

(a) $-2x^2 + 10x + 11$

(b) $-24x^4 - 25x^2 - 28$

(c) $10x^2 + 10x - 11$

(d) $10x^2 + 10x - 3$

**Solution:**

$$\begin{aligned}(4x^2 + 5x - 4) - (-6x^2 - 5x + 7) &= (4x^2 + 5x - 4) + (6x^2 + 5x - 7)\\ &= \boxed{10x^2 + 10x - 11}\end{aligned}$$

**4.** Simplify. $\dfrac{45x^7 - 27x^3 + 36x^5}{-9x^3}$

(a) $-5x^4 + 4x^2$

(b) $-5x^4 + 3 - 4x^2$

(c) $36x^4 - 36 + 27x^2$

(d) $-5x^{21} + 3x^9 - 4x^{15}$

**Solution:**

$$\begin{aligned}\frac{45x^7 - 27x^3 + 36x^5}{-9x^3} &= \frac{45x^7}{-9x^3} + \frac{-27x^3}{-9x^3} + \frac{36x^5}{-9x^3}\\ &= \boxed{-5x^4 + 3 - 4x^2}\end{aligned}$$

**5.** Simplify: $\dfrac{x^4x^{-7}}{x^5}$.

(a) $x^2$

(b) $x^8$

(c) $\dfrac{1}{x^8}$

(d) $\dfrac{1}{x^5}$

**Solution:**

$$\frac{x^4x^{-7}}{x^5} = x^{4+(-7)-5} = x^{-8} = \boxed{\frac{1}{x^8}}.$$

**6.** Factor completely: $4x^2 + 11x - 3$

(a) $(x+3)(4x-1)$

(b) Cannot be factored.

(c) $(x+1)(4x-3)$

(d) $(2x+1)(2x-1)$

**Solution:**

It is a trinomial that is not monic, so use $ac$ method.
We need two numbers $m, n$ so that
$m + n = 11$
$m \cdot n = -12$
12 and $-1$ work. Break the middle term $11x$ as $-x + 12x$ and factor by grouping:

$$\begin{aligned} 4x^2 + 11x - 3 &= 4x^2 - x + 12x - 3 \\ &= x(4x-1) + 3(4x-1) \\ &= \boxed{(4x-1)(x+3)} \end{aligned}$$

**7.** Multiply: $(3x-2)(x^2+4x-5)$

(a) $3x^3 + 10x^2 - 23x + 10$

(b) $3x^3 + 10x^2 + 7x + 10$

(c) $12x^6 - 12x^4 + 10$

(d) $3x^3 - 14x^2 - 23x + 10$

**Solution:**

$$\begin{aligned} &(3x-2)(x^2+4x-5) \\ &= 3x^3 + 12x^2 - 15x - 2x^2 - 8x + 10 \\ &= \boxed{3x^3 + 10x^2 - 23x + 10} \end{aligned}$$

**8.** Divide and write in scientific notation:

$$\frac{3.5 \times 10^7}{5 \times 10^{-5}}$$

(a) $7 \times 10^{10}$

(b) $7 \times 10^{12}$

(c) $0.7 \times 10^{12}$

(d) $7 \times 10^{11}$

**Solution:**

$$\frac{3.5 \times 10^7}{5 \times 10^{-5}} = \frac{3.5}{5} \cdot \frac{10^7}{10^{-5}} = 0.7 \times 10^{12} = 7 \times 10^{11}$$

**9.** Which of the following is a factor of the polynomial $2cx + 5cy - 6dx - 15dy$?

(a) $2x + 5y$

(b) Cannot be factored

(c) $x - 3y$

(d) $c + 3d$

**Solution:**
It has 4 terms, so factor by grouping:

$$\begin{aligned} & 2cx + 5cy - 6dx - 15dy \\ = \; & c(2x + 5y) - 3d(2x + 5y) \\ = \; & (2x + 5y)(c - 3d) \end{aligned}$$

Therefore the answer is $\boxed{\text{2x+5y}}$

**10.** Expand: $(a + b)^2$

(a) $a^2 + 2ab + b^2$

(b) $a^2 + b^2$

(c) $(a + b)(a - b)$

(d) $a^2 - b^2$

**Solution:** $(a + b)^2 = \boxed{a^2 + 2ab + b^2}$, as you should have memorized.

**11.** Factor: $4x^2 - 25$.

(a) $(2x + 5)(2x - 5)$

(b) $(2x - 5)^2$

(c) Cannot be factored.

(d) $2(x - 5)(x + 5)$

**Solution:**
It has 2 terms, so if it can be factored it is because it is a difference of squares. It is: the first term is $(2x)^2$ and the second is $5^2$. Therefore $4x^2 - 25 = \boxed{(2x + 5)(2x - 5)}$.

**12.** Which of the following is a factor of $4x^4 - 100x^2$?

(a) $4x - 10$

(b) $10$

(c) $x^2 + 5$

(d) $x + 5$

**Solution:**
Factor out the GCF: the GCF of the coefficients is 4. The GCF for $x$ is $x^2$.
Therefore the GCF is $4x^2$:
$4x^4 - 100x^2 = 4x^2(x^2 - 25)$.
The first two factors are monomials, so they cannot be factored further.
The last term $(x^2-25)$ is a difference of squares, which is factored as $(x + 5)(x - 5)$.
Therefore $4x^4 - 100x^2 = 4x^2(x + 5)(x - 5)$.
The only factor that appears as solution is $\boxed{x + 5}$.

**13.** Give the product in scientific notation.

$$(6 \times 10^3)(7 \times 10^7)$$

(a) $4.2 \times 10^{11}$

(b) $42 \times 10^{10}$

(c) $4.2 \times 10^9$

(d) $4.2 \times 10^{10}$

**Solution:**

$$\begin{aligned}(6 \times 10^3)(7 \times 10^7) &= 42 \times 10^{3+7} \\ &= 42 \times 10^{10} \\ &= 4.2 \times 10^{11}\end{aligned}$$

**14.** Write with only positive exponents:

$$\left(\frac{12x^2y^{-3}}{4x^{-5}}\right)^{-2}$$

(a) $\dfrac{y^6}{9x^{14}}$

(b) $-\dfrac{6x^6}{y^6}$

(c) $\dfrac{9y^6}{x^9}$

(d) $-9y^6x^{-6}$

**Solution:**

$$\begin{aligned}\left(\frac{12x^2y^{-3}}{4x^{-5}}\right)^{-2} &= \left(3x^{2-(-5)}y^{-3}\right)^{-2} \\ &= \left(3x^7y^{-3}\right)^{-2} \\ &= 3^{-2}x^{-14}y^6 \\ &= \boxed{\frac{y^6}{9x^{14}}}\end{aligned}$$

Free response questions start here. SHOW ALL WORK!!!

**15.** Factor completely: $x^6y^3 - 16x^2y^7$

**Solution:**

Factor the common factors first. Then factor the binomial as a difference of squares. Finally, one of the factors is also a difference of squares, so it can be factored:

$$\begin{aligned}&x^6y^3 - 16x^2y^7 \\ &= x^2y^3(x^4 - 16y^4) \\ &= x^2y^3(x^2 + 4y^2)(x^2 - 4y^2) \\ &= x^2y^3(x^2 + 4y^2)(x + 2y)(x - 2y)\end{aligned}$$

**16.** Multiply: $(7x - 5)(7x + 5)$

**Solution:**

Use the formula $(a - b)(a + b) = a^2 - b^2$:

$$\begin{aligned}(7x - 5)(7x + 5) &= (7x)^2 - 5^2 \\ &= 49x^2 - 25\end{aligned}$$

**17.** Factor completely: $3x^3 - 15x^2 + 18x$.

**Solution:**

Factor out the common factors and then factor the trinomial:

$$\begin{aligned} 3x^3 - 15x^2 + 18x &= 3x(x^2 - 5x + 6) \\ &= 3x(x-2)(x-3) \end{aligned}$$

**18.** Multiply: $(x^2 + 3x - 6)(x - 7)$

**Solution:**

$$\begin{aligned} &(x^2 + 3x - 6)(x - 7) \\ &= x^3 - 7x^2 + 3x^2 - 21x - 6x + 42 \\ &= \boxed{x^3 - 4x^2 - 27x + 42}. \end{aligned}$$

**19.** Divide: $\dfrac{9x^3 - 6x^2}{3x^2}$.

**Solution:**

$$\begin{aligned} \frac{9x^3 - 6x^2}{3x^2} &= \frac{9x^3}{3x^2} + \frac{-6x^2}{3x^2} \\ &= 3x - 2. \end{aligned}$$