MTH 05, Test 3, V. 1, 11/21/17 Luis Fernández

NAME: SOLUTION

There are twenty-two questions, each worth 5 points. For multiple-choice questions, circle your answer. For free-response questions, SHOW ALL WORK to receive full credit.

1. Multiply: $(4x-5)(x^2-3x+2)$

(a) $4x^3-12x^2+23x-10$

(b) $4x^3-17x^2-7x-10$

(c) $4x^3-12x^2-7x-10$

(d) $4x^3-17x^2+23x-10$

2. Divide and write in scientific notation:

$$\frac{3.6\times10^{13}}{4\times10^{7}}$$

(a) 9×10^6

(b) 9×10^7

(c) 0.9×10^6

(d) 9×10^5

3. Write using only positive exponents:

$$(-x^3y^{-6}z^5)(8x^{-3}yz^4)$$

(a) $-\dfrac{8z^{20}}{x^9y^6}$

(b) $\dfrac{z^9}{8y^5}$

(c) $-\dfrac{8z^9}{y^5}$

(d) $\dfrac{24x^6z^9}{y^5}$

4. Which of the following is a factor of the polynomial: $x^2+11x+30$?

(a) $(x+6)$

(b) $(x+11)$

(c) $(x-6)$

(d) $(x-5)$

5. Simplify: $(4x^2 - 7x + 9) - (-2x^2 - 2x + 3)$.

(a) $2x^2 + 5x + 6$

(b) $6x^2 - 9x + 12$

(c) $6x^2 - 5x + 6$

(d) $2x^2 - 9x + 12$

6. Simplify: $\dfrac{21x^3 - 28x^2 + 7x}{-7x}$

(a) $-3x^2 + 4x - 1$

(b) $-3x^4 + 4x^3 - x^2$

(c) $21x^3 - 28x^2$

(d) $-3x^2 + 4x$

7. Simplify: $\dfrac{x^2 x^{-4}}{x^3}$.

(a) x^3

(b) $\dfrac{1}{x^5}$

(c) $\dfrac{1}{x^3}$

(d) x^5

8. Which of the following is a factor of the polynomial: $2cx - 5cy - 6dx + 15dy$?

(a) $2x + 5y$

(b) $2x - 5y$

(c) $x - 3y$

(d) $c + 3d$

9. Factor: $x^2 - 9$.

(a) $(x+3)^2$

(b) $(x+3)(x-3)$

(c) Cannot be factored.

(d) $(x-9)^2$

10. Expand: $(a+b)^2$

(a) $a^2 + 2ab + b^2$

(b) $a^2 - b^2$

(c) $(a+b)(a-b)$

(d) $a^2 + b^2$

11. The solutions of the equation $x^2 - 9x - 22 = 0$ are:

(a) 2 and -11

(b) -2 and 11

(c) It has no solutions.

(d) -9 and -22

12. Write with only positive exponents:

$$\left(\frac{12x^2y^{-3}}{4x^{-5}}\right)^{-2}$$

(a) $\dfrac{y^6}{9x^{14}}$

(b) $-9y^6x^{-6}$

(c) $\dfrac{9y^6}{x^9}$

(d) $-\dfrac{6x^6}{y^6}$

13. Which of the following is a factor of $3x^3 - 12x$?

(a) $x - 2$

(b) $x - 3$

(c) $x - 4$

(d) 12

14. Multiply: $(3x + 5)(3x - 5)$

(a) $9x^2 + 30x + 25$

(b) $9x^2 - 25$

(c) $6x^2 + 25$

(d) $6x^2 - 30x + 25$

15. Factor completely: $x^2 - 8x - 20$

(a) $(x - 10)(x + 2)$

(b) $(x - 8)(x - 20)$

(c) $(x + 10)(x - 2)$

(d) $(x - 8)(x + 2)$

16. The solutions of the equation $(x-3)(x+1) = 0$ are

(a) It has no solutions

(b) 2 and -4

(c) 3 and -1

(d) -3 and 1

Free response questions start here. SHOW ALL WORK!!!

17. Solve the equation $3x^2 + 8x + 5 = 0$.

Solution:

Factor the polynomial on the LHS of the equation. Use the ac-method: first find m and n such that $m + n = 8$ and $m \cdot n = 15$. This is not hard: 3 and 5. Then write the $8x$ as $5x + 3x$ and factor by grouping:

$$3x^2 + 8x + 5 = 0$$
$$3x^2 + 5x + 3x + 5 = 0$$
$$x(3x + 5) + (3x + 5) = 0$$
$$(3x + 5)(x + 1) = 0$$

Therefore $(3x + 5) = 0$ or $(x + 1) = 0$, which gives

$$x = -\frac{5}{3} \quad \text{or} \quad x = -1.$$

Therefore the solutions are $-\frac{5}{3}$ and -1.

18. Factor completely: $3x^3 - 15x^2 + 18x$.

Solution:

Factor out the common factors and then factor the trinomial:

$$\begin{aligned} 3x^3 - 15x^2 + 18x &= 3x(x^2 - 5x + 6) \\ &= 3x(x-2)(x-3) \end{aligned}$$

19. Multiply: $(6x - 3)(6x + 3)$

Solution:

Use the formula $(a - b)(a + b) = a^2 - b^2$:

$$\begin{aligned} (6x-3)(6x+3) &= (6x)^2 - 3^2 \\ &= 36x^2 - 9 \end{aligned}$$

20. Write the following in simplest radical form:

a) $\sqrt{18}$ b) $\sqrt{72}$

Solution:

Let us write each root in simplest radical form:

a) $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$.
b) $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$.

21. A **positive** number is 9 more than another. The product of the two numbers is 52. What are the numbers?

Solution:

Suppose that the smaller number is called x. Then the greater will be $(x + 9)$. Their product is 52, so we get the equation $x(x + 9) = 52$. To solve it, first expand the LHS and then move the 52 to the LHS:

$$x(x + 9) = 52$$
$$x^2 + 9x = 52$$
$$x^2 + 9x - 52 = 0$$

Now factor the LHS to get the equation

$$(x + 13)(x - 4) = 0.$$

This implies $(x + 13) = 0$ or $(x - 4) = 0$.
Therefore $x = -13$ or $x = 4$.
Since the numbers are positive, only the solution $x = 4$ works. x is what we called the smaller number. The other one is therefore $4 + 9 = 13$. Therefore the two numbers are 4 and 13.

22. Factor completely: $x^4y^3 - 4x^2y^5$

Solution:

Factor the common factors first. Then factor the binomial as a difference of squares:

$$\begin{aligned} x^4y^3 - 4x^2y^5 &= x^2y^3(x^2 - 4y^2) \\ &= x^2y^3(x + 2y)(x - 2y) \end{aligned}$$