CSI 35: Discrete Mathematics II. Midterm 2

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Print Name: SOLUTION

INSTRUCTIONS:

- This exam contains 16 questions, 6 pages for a total of 106 points (points over 100 count as extra credit).
- You have 110 minutes to complete the exam.
- **You must show all your work** in order to get credit.
- You can use a non-graphing scientific calculator. No other electronic devices, notes or books are permitted.

Part I: Fill in the blanks. (4 points each)

1. A **relation** on a set A is a subset of $A \times A$.
2. A relation is an **equivalence relation** if it is reflexive, symmetric, and transitive.
3. A relation R on a set A is **reflexive** if for all $x \in A$, $(x, x) \in R$
4. A relation R on a set A is **symmetric** if for all $x, y \in A$, $(x, y) \in R \implies (y, x) \in R$.
5. A relation R on a set A is antisymmetric if $\forall x, y \in A$, $[(x, y) \in R \wedge (y, x) \in R] \implies x = y$.
6. A relation R on a set A is **transitive** if

   $\forall x, y, z \in A$, $(x, y) \in R$ and $(y, z) \in R$ $\implies (x, z) \in R$.
7. A relation on a set A is a **partial order** if it is reflexive, antisymmetric, and transitive.
8. An element x in a poset $(A, \preceq)$ is a minimal element if $\forall y \in A$, $y \preceq x \implies y = x$.
9. Two elements x, y in a poset $(A, \preceq)$ are called **comparable** if $x \preceq y$ or $y \preceq x$

Part II: Write answers in spaces provided (10 points each)

10. Give an example of a symmetric relation on the set $\{a, b, c, d\}$ which is not reflexive. (List the pairs in your relation.)

For example
$R = \{(a,b), (b,a)\}$.

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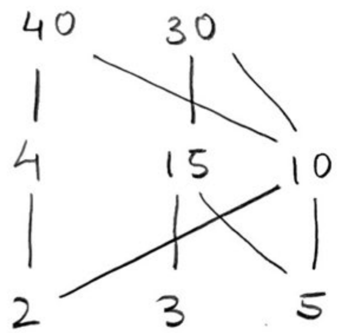
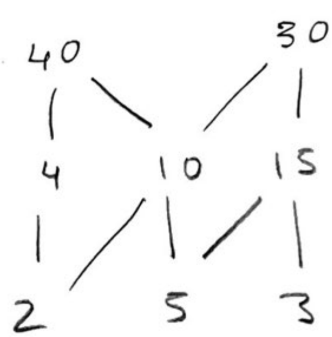
11. Consider the poset $(\{2, 3, 4, 5, 10, 15, 30, 40\}, |)$.

(a) Draw the Hasse diagram.



or



(b) What are the minimal elements, if any? What are the maximal elements, if any?

Minimal: 2, 5, 3

Maximal 30, 40

13. Below is the zero-one matrix representating a relation R on the ordered set $\{a, b, c, d\}$:

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) List all the pairs in this relation.

$$\boxed{R = \{(a,b), (a,d), (b,a), (b,b), (c,b), (c,d), (d,d)\}}$$

(b) Explain why is R not symmetric.

Because $(a,b) \in R$ but $(b,a) \notin R$

(c) List all the pairs of the relation $R^2 = R \circ R$.

$R = \{(a,a), (a,b), (a,d), (b,b), (b,d), (b,a), (c,a), (c,b),$
$(c,d), (d,d)\}$

12. Let R be the (mod 5) relation on the integers (that is, $(x, y) \in R$ if 5 divides $x - y$).

(a) List 6 elements of each of the equivalence classes $[1]_5$, $[7]_5$, $[23]_5$, $[4]_5$, and $[6]_5$.

$[1]_5 = \{\ldots, 1, 6, 11, 16, 21, 26, \ldots\}$

$[7]_5 = \{\ldots, 2, 7, 12, 17, 22, 27, \ldots\}$

$[23]_5 = \{\ldots, 3, 8, 13, 18, 23, 28, \ldots\}$

$[4]_5 = \{\ldots, 4, 9, 14, 19, 24, 29, \ldots\}$

$[6]_5 = \{\ldots, 1, 6, 11, 16, 21, 26, \ldots\}$

(b) Do the equivalence classes $[1]_5$, $[7]_5$, $[23]_5$, $[4]_5$ and $[6]_5$ form a partition of $\mathbb{Z}$? If not, which equivalence class is missing?

No : ..., 0, 5, 10, ... are missing. this is

$\boxed{[0]_5}$.

(turn over)

14. In the poset $(\mathcal{P}(\{1,2,3,4,5\}), \subseteq)$, where $\mathcal{P}(S)$ denotes the power set of S,

(a) Are the sets $\{1,3,5\}$ and $\{1,3,4,5\}$ comparable? If so, how are they related in the partial order?

Yes, because $\{1,3,5\} \subseteq \{1,3,4,5\}$

(b) Give a pair of incomparable subsets.

$\{1,3,5\}$ and $\{2,3,4\}$
(neither contains the other).

(c) Is this poset a total order? Why?

No, because there are incomparable elements.

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15. In the lexicographic order on $\mathbb{Z} \times \mathbb{Z}$, find integers n, m such that

(a) $(-3, m) \preceq (-3, 2)$

$m = 1$, for exaple

(b) $(2, -2) \preceq (2, n)$.

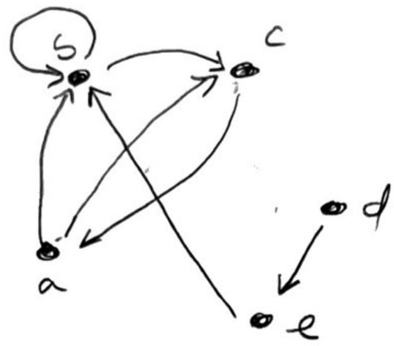
$n = 0$, for exaple

(turn over)

16. .2in Consider the relation on the set $\{a, b, c, d, e\}$ given by

$$R = \{(a, b), (a, c), (b, b), (b, c), (c, a), (d, e), (e, b)\},$$

(a) Draw a directed graph which represents the relation.



(b) Write the zero-one matrix of the relation.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(c) Is the relation transitive? If not, explain why.

No, because for exaple,
$(e, b) \in R$ and $(b, c) \in R$,
but $(e, c) \notin R$