

CSI 35: Discrete Mathematics II. Midterm 1, Version B

Professor Luis Fernández

NAME: _____

INSTRUCTIONS:

- This exam contains 9 questions, 6 pages for a total of 104 points (points over 100 count as extra credit).
 - You have 110 minutes to complete the exam.
 - **You must show all your work** in order to get credit.
 - You can use a non-graphing scientific calculator. No other electronic devices, notes or books are permitted.
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1. [10 points] We recursively define a function f from the positive integers to the integers as follows: $f(1) = -1$ and $f(n) = 2 \cdot f(n-1) + 5$.

Evaluate the following.

(a) $f(1)$

(b) $f(2)$

(c) $f(3)$

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2. [12 points] Let $F(n) = -4n$, defined on integers $n \geq 1$. Find a *recursive* definition for F .

3. [12 points] Consider the following program:

```
if  $x < 0$  :  
     $x = x + 3$   
else:  
     $x = 2x + 1$ 
```

Suppose you are given that x is initialized to some positive integer. Show that after running the program, the value of x is odd.

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4. [12 points] Give a recursive definition of the set of all positive integers that are multiples of 5.

5. [10 points] Consider the proposition: 3 divides $n^3 - n$ for $n \geq 2$. Suppose that we want to prove this proposition by induction.
- (a) State the propositional function $P(n)$ that you want to prove.
 - (b) State and prove the BASIC STEP.
 - (c) State the INDUCTIVE STEP. **Do not prove it.**
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6. [12 points] Write a recursive algorithm called “ $a(n)$ ”, for n a nonnegative integer, that computes the n^{th} term of the sequence a_n defined by $a_0 = 1$, $a_1 = 3$, and $a_n = 3a_{n-1} + a_{n-2}$. You should write it as follows:

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procedure a(n: n nonnegative integer)

    if n = ...

    else if ...

    else ...
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7. [12 points] Let r be a real number. Prove by induction that $r^0 + r^1 + r^2 + \dots + r^n = \frac{r^{n+1} - 1}{r - 1}$.
Make sure to clearly state $P(n)$ and clearly explain the base and inductive steps.

8. [12 points] Consider the set S which is defined recursively as follows:

$3 \in S$ and $5 \in S$

If $x \in S$ and $y \in S$, then $x + y \in S$.

(a) Is 7 in S ? Is 11 in S ?

(b) Find all the positive integers that are **not** in S .

9. [12 points] Consider the statement

$P(n)$: “every amount of postage of 20 cents or more can be formed using just 5-cent and 6-cent stamps”.

Prove this proposition using strong induction by completing the BASIC STEP outlined below and doing the INDUCTIVE STEP.

- BASIC STEP: $P(20)$, $P(21)$, $P(22)$, $P(23)$, and $P(24)$ are true. [You need to prove each base case.]

- INDUCTIVE STEP: (do this step completely)