CSI 35: Discrete Mathematics II. Midterm 1

Professor Luis Fernández

NAME: ____

INSTRUCTIONS:

- This exam contains 9 questions, 5 pages for a total of 106 points (points over 100 count as extra credit).
- You have 110 minutes to complete the exam.
- **You must show all your work** in order to get credit.
- You can use a non-graphing scientific calculator. No other electronic devices, notes or books are permitted.

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1. [12 points] We recursively define f as follows: $f(1) = 2$ and $f(n) = 5 \cdot f(n-1) + 10$.

   Evaluate the following.

   1. $f(1)$
   2. $f(2)$
   3. $f(3)$

2. [12 points] Let $F(n) = 5n$, defined on integers $n \geq 1$. Find a recursive definition for F.

3. [12 points] Consider the following program:

   if $x < 0:$
   $\quad x = x + 7$
   else:
   $\quad x = 6x$

   Suppose you are given that x is initialized to some positive integer. Show that after running the program, the value of x is even.

4. [12 points] Give a recursive definition of the set of all positive integers that are multiples of 7.

5. [10 points] Consider the proposition: 5 divides $n^5 - n$ for $n \geq 2$.

   In order to prove this by induction you should (1) State what statement $P(n)$ you are doing induction on, (2) Do the base step: show $P(n)$ is true for the base case, and (3) Do the inductive step.

   Just do (1) and (2). That is, state exactly what $P(n)$ is, then state and prove the base case. Do **not** do the inductive step.

6. [12 points] Prove by induction that $2 + 4 + 6 + \cdots + 2n = n(n+1)$.

   (That is, the sum of the first n positive even integers is $n(n+1)$.)

7. [12 points] Devise a recursive algorithm called "$a(n)$", for n is a nonnegative integer, that computes the n^{th} term of the sequence a_n defined by $a_0 = 1$, $a_1 = 2$, and $a_n = a_{n-1} \cdot a_{n-2}$.

8. [12 points] Consider the set S which is defined recursively as follows:

$2 \in S$ and $5 \in S$
If $x \in S$ and $y \in S$, then $x + y \in S$.

1. Is 20 in S? Is 13 in S?
2. Find all the positive integers that are **not** in S.

9. [12 points] Prove that every amount of postage of 6 cents or more can be formed using just 3-cent and 4-cent stamps. Use strong induction with the proof starting as follows:

   - Let $P(n)$ be "postage of n cents can be formed using just 3-cent and 4-cent stamps."
   - Base cases: $P(6), P(7)$, and $P(8)$. [You need to state and prove each base case]

   Now finish the proof by doing the inductive step.