Chapter 11. Trees

11.1 Introduction to trees

Definition 1:

- A **tree** is an undirected graph that is connected and has no simple circuits.
- A **forest** is an undirected graph that has no simple circuits (but it may be disconnected).

Theorem 1: An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

Definition 2:

- A **rooted tree** is a tree in which one vertex has been *designed* as the root and every edge is directed away from the root.

Terminology: In a rooted tree...

- If v is a vertex other than the root, the **parent** of v is the unique vertex u that is connected to v and such that the path from the root to v passes through u.
- If u is the parent of v then v is a **child** of u.
- Vertices with the same parents are called **siblings**.
- The **ancestors** of a vertex are the vertices lying on the path from the root to this vertex. The root has no ancestors.
- The **descendants** of a vertex are those vertices that have v as an ancestor.
- A vertex of a rooted tree is called a **leaf** if it has no children.
- Vertices that have children are called **internal vertices.**
- If a is a vertex in a tree, then the subtree with a as the root is the subgraph consisting of a and its descendants.

Definition 3:

- A rooted tree is called an ***m*-ary tree** if every internal vertex has at most m children.
- It is called a **full *m*-ary tree** if every internal vertex has *exactly* m children.
- An m-ary tree with m=2 is called a **binary tree**.

Definition 4:

- An **ordered rooted tree** is a rooted tree where the children of each internal vertex are ordered. They are drawn so that the children of each vertex are ordered left to right.
- In an ordered rooted binary tree, if a vertex has two children, they are called the **left** and the **right** children of the vertex.
- The tree rooted at the left child is called the **left subtree**. The tree rooted at the right child is called the **right subtree**.

Definition 5:

- The **level** of a vertex in a rooted tree is the length of the path connecting that vertex and the root. The level of the root is defined to be 0.
- The **height** of a rooted tree is the maximum level of all its vertices.

Exercises:

- Draw all non-isomorphic trees with four vertices. Do not label the vertices of the graph. You should not include two trees that are isomorphic.
- Draw all non-isomorphic *rooted* trees with four vertices. Do not label the vertices of the graph. You should not include two trees that are isomorphic.
- If a tree has n vertices, how many edges does it have?
- If a full m-ary tree has i internal vertices, how many vertices does it have in total? How many leaves?
- How many leaves, at most, has an m-ary tree with height h?