MATH 71000 - Differential Geometry I (35178)

Professor: Luis Fernández
Office & Tel.: Rm. 4213, (212)-817-8561.
Class times and room: Mo, We, 10:00 to 11:30, Rm 6494.
Office hours: We 12-1, or by appointment.
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Class Plan

- Each class will start with questions about the previous material and exercises.
- Students are expected to read the section assigned for each day of class in the reference given or elsewhere (please see the syllabus below) and try to solve the first few exercises of that chapter.
- Please, please, ASK QUESTIONS. Anything you ask is great, and I will answer in or out of class.
- In my opinion, the best way to understand geometry is by understanding many examples. Sometimes the computations will seem far too long. In this case, you are very encouraged to use a computer algebra program (Mathematica, Maple, etc).

Homework, Tests, etc

- Homework will be assigned each week. It is due on Wednesday of the week after.
- On Mondays we will spend some time discussing the homework assignment, as well as answering any questions that there may arise.
- Some randomly chosen exercises will be graded.
- The final exam for this class will be the Qualifying Exam in Differential Geometry, which is scheduled for Friday, May 26th, 10:00am - 1:00pm. There will be no other tests.
- Qualifying Exam samples can be found at the Math Department website, or by following the link in the course webpage above.

References

- [W]: *Foundations of Differentiable Manifolds and Lie Groups*, by Frank W. Warner.
- [S1]: *A Comprehensive Introduction to Differential Geometry, Vol 1*, by Michael Spivak.
- [S2]: *A Comprehensive Introduction to Differential Geometry, Vol 2*, by Michael Spivak.
- [S3]: *A Comprehensive Introduction to Differential Geometry, Vol 3*, by Michael Spivak.
- [DC] *Riemannian Geometry*, by Manfredo P. DoCarmo.
- [L1] *Manifolds and Differential Geometry*, by John M. Lee.
- [L2] *Riemannian Manifolds. An introduction to curvature*, by John M. Lee.

Syllabus

Of course, this is just a tentative plan. It will most probably change as we advance, and it will be updated accordingly.

DATE	SECTION	REFERENCE
Mo 1/30	Review of the first part of the course: Manifolds, differentiable maps, tangent and cotangent bundles, vector fields, flow, differential forms and operations with them (exterior derivative, Lie derivative, interior product by vector fields), orientation, Poincaré Lemma. Many examples: $\mathbb{R}^n$, S^n, $\mathbb{RP}^n$, $\mathrm{Gr}(k, \mathbb{R}^n)$, Lie Groups, $\mathbb{CP}^n$, $\mathrm{Gr}(k, \mathbb{C}^n)$, Homogeneous Spaces	[W], [L1]
We 2/1	Integration and Stokes' Theorem.	[W] Chapter 4.
Mo 2/6	Integration and Stokes' Theorem.	[W] Chapter 4.
We 2/8	Vector bundles and operations with them. Fiber bundles.	[L1] pp. 257–262 & 270–277.
Mo 2/13	NO CLASS - LINCOLN'S BIRTHDAY	
We 2/15	Riemannian Metrics	[DC] Chapter 1.
Mo 2/20	NO CLASS - PRESIDENT'S DAY.	
We 2/22	Riemannian Metrics	[DC] Chapter 1.
Mo 2/27	Affine Connections	[DC] Chapter 2.
We 3/1	Affine Connections	[DC] Chapter 2.
Mo 3/6	Affine Connections	[DC] Chapter 2.
We 3/8	Geodesics	[DC] Chapter 3.
Mo 3/13	Geodesics	[DC] Chapter 3.
We 3/15	Convex neighbourhoods	[DC] Chapter 3.
Mo 3/20	Curvature	[DC] Chapter 4.
We 3/22	Curvature	[DC] Chapter 4.
Mo 3/27	Curvature	[DC] Chapter 4.
We 3/29	An alternative view: Moving Frames	[Sp2], Chapter 7.
Mo 4/3	Particular case of surfaces in $\mathbb{R}^3$. Gauss-Bonnet Theorem	[S3], Chapter 6.
We 4/5	Particular case of surfaces in $\mathbb{R}^3$. Gauss-Bonnet Theorem	[S3], Chapter 6.
Mo 4/10	NO CLASS - SPRING RECESS.	
We 4/12	NO CLASS - SPRING RECESS.	
Mo 4/17	NO CLASS - SPRING RECESS.	
We 4/19	Hopf-Rinow Theorem	[DC] Chapter 7.
Th 4/20	Hadamard's Theorem	[DC] Chapter 7.
Mo 4/24	Jacobi Fields	[DC] Chapter 5.
We 4/26	Jacobi Fields	[DC] Chapter 5.
Mo 5/1	Isometric Immersions	[DC] Chapter 6.
We 5/3	Isometric Immersions	[DC] Chapter 6.
Mo 5/8	Spaces of Constant Curvature	[DC] Chapter 8.
We 5/10	Spaces of Constant Curvature	[DC] Chapter 8.
Mo 5/15	Introduction to Harmonic Maps	TBD
We 5/17	Introduction to Harmonic Maps	TBD