MATH 33 - Analytic Geometry and Calculus III, Sec. E01 – 20088

Third test. Time allowed: two hours. Professor Luis Fernández

NAME:____

INSTRUCTIONS: Solve the following exercises. **You must show work** and **justify your answers** in order to receive credit in any of the exercises.

[14] **1.** Consider the curve $\vec{r}(t) = (2t, 1-3t, 5+4t)$.

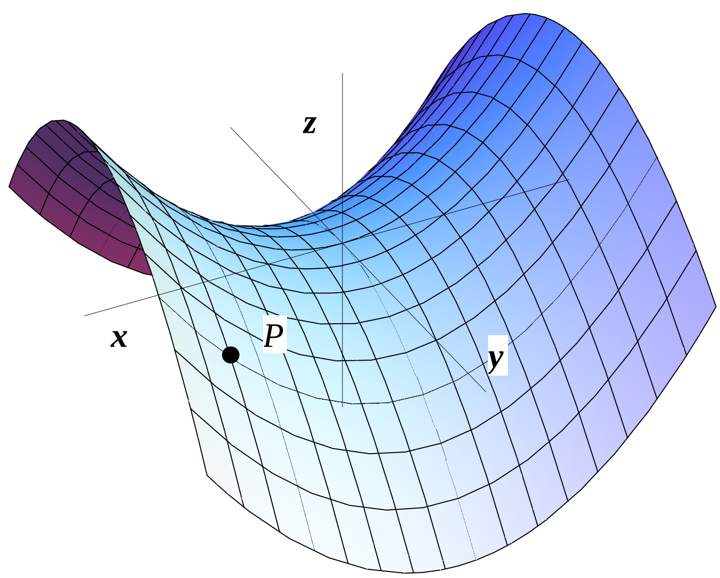
a) Find the length of the curve $\vec{r}$ from $t=2$ to $t=5$.

b) Reparametrize the curve $\vec{r}$ with respect to arc length.

c) Find the curvature of $\vec{r}$. (Remember that $\kappa(s) = \frac{d\vec{T}}{ds}$, where $\vec{T}(s)$ is the unit tangent vector to $\vec{r}(s)$.)

---

[10] **2.** Determine the signs of the partial derivatives f_x and f_y at the point P for the function f whose graph is shown.



---

[12] **3.** Find the limit, if it exists, or show that the limit does not exist.

a) $\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4+3y^4}$.

b) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$ (HINT: Write x and y in polar coordinates (r,θ) and do the limit when $r \to 0^+$.)

---

[10] **4.** Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions. You do not need to simplify the answer.

a) $f(x,y) = xye^{xy}$

b) $f(x,y) = (2x+3y)^{10}$

[10] **5.** For $f(x, y) = x^3y^5 + 2x^4y$, find

a) $\dfrac{\partial^2 f}{\partial x^2}$

b) $\dfrac{\partial^2 f}{\partial x \partial y}$

c) $\dfrac{\partial^2 f}{\partial y \partial x}$

d) $\dfrac{\partial^2 f}{\partial y^2}$

---

[12] **6.** For the function $f(x, y) = x\sqrt{y}$,

a) Find the linearization $L(x, y)$ at the point $(1, 4)$.

b) Find the equation of the tangent plane to the graph of f at the point $(1, 4, 2)$.

---

[12] **7.** If $z = f(x, y)$, where f is differentiable, and

- $x = g(t)$ and $y = h(t)$.
- $g(3) = 2$ and $h(3) = 7$.
- $g'(3) = 5$ and $h'(3) = -4$.
- $f_x(2, 7) = 6$ and $f_y(2, 7) = -8$.

Find $\dfrac{dz}{dt}$ when $t = 3$.

---

[10] **8.** Let $R = \text{Ln}(u^2 + v^2 + w^2)$, where $u = x + 2y$, $v = 2x - y$, and $w = 2xy$.

Find $\dfrac{\partial R}{\partial x}$ and $\dfrac{\partial R}{\partial y}$ when $x = 1$, $y = 1$.

---

[10] **9.** For the function $f(x, y) = \sin(2x + 3y)$

a) Find the gradient of f at the point $(-6, 4)$.

b) Find $D_{\vec{u}}f(-6, 4)$, where $\vec{u} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

---

[10] **10.** Find the maximum rate of change of $f(x, y) = \dfrac{y^2}{x}$ at the point $(2, 4)$ and the direction at which it occurs.