

MATH 33 - Analytic Geometry and Calculus III, Sec. E01 – 20088

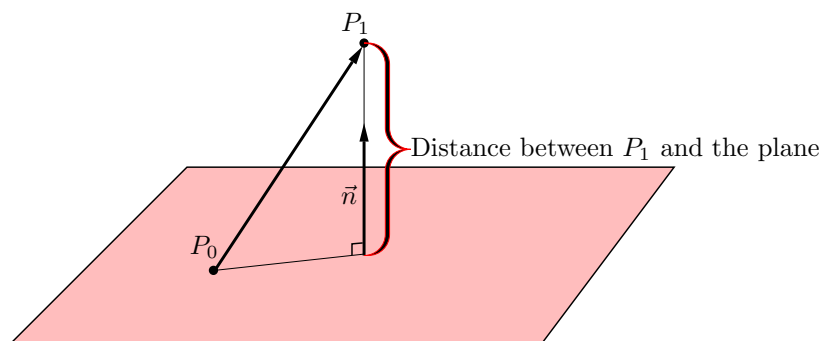
Second test. Time allowed: two hours. Professor Luis Fernández

NAME: \_\_\_\_\_

**INSTRUCTIONS:** Solve the following exercises. **You must show work** and **justify your answers** in order to receive credit in any of the exercises.

- [12] 1. For the power series  $\sum_{n=1}^{\infty} \frac{x^n}{n 2^n}$ ,
- Find the radius of convergence.
  - Find the interval of convergence.
- [12] 2. Find the Taylor series of the function  $f(x) = \cos x$  at the point  $a = \frac{\pi}{2}$ .
- [14] 3. Given  $f(x) = \sqrt{x}$ ,
- Approximate the function  $f$  by a Taylor polynomial with degree 2 at the number  $a = 4$ .
  - Use Taylor's inequality to estimate the accuracy of the approximation  $f(x) \approx T_2(x)$  when  $x$  lies in the interval  $4 \leq x \leq 5$ .
- [12] 4. Use the binomial series to expand the function  $f(x) = \frac{1}{(x+1)^2}$  as a power series.
- [10] 5. Find the equation of a sphere centered at the point  $O = (3, -2, 4)$  that passes through the point  $P = (5, -1, 2)$ .
- [15] 6. Let  $A = (2, 1, 3)$ ,  $B = (3, 2, 4)$  and  $C = (1, 2, 2)$  be the vertices of a triangle. Let  $\vec{a}$  be the vector from  $A$  to  $B$  and let  $\vec{b}$  be the vector from  $B$  to  $C$ .
- Calculate the coordinates of the vectors  $\vec{a}$  and  $\vec{b}$ .
  - Find the length of  $\vec{a}$  and the length of  $\vec{b}$ .
  - Find the angle between  $\vec{a}$  and  $\vec{b}$  (you may need to use your calculator).
  - Find the projection  $\text{proj}_{\vec{a}} \vec{b}$  of  $\vec{b}$  over  $\vec{a}$ .
  - Calculate the area of the triangle  $\triangle ABC$ .
- [9] 7. Let  $\vec{a} = (2, 1, -4)$ ,  $\vec{b} = (1, -1, -3)$ , and  $\vec{c} = (3, -2, -3)$ .
- Find  $\vec{a} \times \vec{b}$
  - Find a unit vector that is perpendicular to both  $\vec{a}$  and  $\vec{b}$ .
  - Find the volume of the parallelepiped determined by  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$ .
- [15] 8. Given  $P = (2, -3, 4)$ ,  $Q = (1, 1, -3)$ , and  $\vec{n} = (1, -2, -1)$ ,
- Find the equation of the plane that passes through the point  $P$  and is perpendicular to the vector  $\vec{n}$ .
  - Find the equation of the line perpendicular to the plane of part a) that passes through the point  $Q$ .
  - Find the equation of the plane that passes through the points  $P$ ,  $Q$  and  $R = (0, 1, 1)$ .

- [10] 9. Use the following picture to find the distance between the plane  $2(x - 1) - 3(y + 1) + (z - 2) = 0$  and the point  $P = (2, 3, 4)$ .



**HINT:** Draw the projection of  $\overrightarrow{P_0P_1}$  over  $\vec{n}$ .