Ratios and rates problems

Remember: In all these problems the goal is to write a proportion and then solve it. Each quantity in the problem will always be related to two other quantities. The only thing to remember is that related quantities must appear either in the same fraction or at the same position in the two fractions of the proportion.

Example: An ice cream factory makes 160 quarts of ice cream in 2 hours. How many quarts could be made in 12 hours?

The unknown quantity (let us call it x) is the number of quarts that can be made in 12 hours.

The quantity 160 quarts is related to:

→ The 2 hours it takes to make it.

→ The unknown amount of quarts x (because they are both 'quarts').

It is easiest to do a diagram:

$$\begin{array}{ccc} 160 \text{ quarts} & \longleftrightarrow & 2 \text{ hours} \\ \updownarrow & & \updownarrow \\ x \text{ quarts} & \longleftrightarrow & 12 \text{ hours} \end{array}$$

(The top horizontal arrow expresses that to make 160 quarts one needs 2 hours; the vertical arrows express the quantities that have the same units.)

Once we have this the rest is easy: write the proportion: $\dfrac{160}{x} = \dfrac{2}{12}$, and solve it: $2 \cdot x = 160 \cdot 12$, so $2x = 1920$, which after dividing both sides by 2 gives $x = 960$. Therefore, in 12 hours the ice cream factory would make 960 quarts of ice cream.

Example: You can buy 3 apples at the Quick Market for $1.23. You can buy 5 of the same apples at Stop and Save for $2.45. Which place is the better buy?

To compare the deals, we need to find the price of a single apple at each place.

→ If 3 apples at Quick Market cost $1.23, that means that each apple costs $\frac{\$1.23}{3} = 41$ cents.

→ If 5 apples at Stop and Save cost $2.45, that means that each apple costs $\frac{\$2.45}{5} = 49$ cents. Therefore 3 apples at Quick Market is a better deal.

Example: In a sample of 600 bottles, 11 were found to be leaking. Approximately how many bottles would you expect to be leaking in a sample of 21,000 bottles?

The quantity of 600 bottles in the sample is related to two quantities:

→ The 11 bottles out of these 600 that were leaking.

→ The 21,000 bottles because they are also a sample. It is easiest to do a diagram:

$$\begin{array}{ccc} 600 \text{ bottles in sample} & \longleftrightarrow & 11 \text{ leaking bottles} \\ \updownarrow & & \updownarrow \\ 21,000 \text{ bottles in sample} & \longleftrightarrow & x \text{ leaking bottles} \end{array}$$

Write the proportion: $\dfrac{600}{21000} = \dfrac{11}{x}$, and solve it: $600 \cdot x = 11 \cdot 21000$, so $600x = 21000$, and divide both sides by 600 to get $x = 385$.

<u>Exercises</u>

1. If eight trash bags last for thirty days, then twenty-eight trash bags will last for how many days?

2. If four out of ten people vote, then in a town with two hundred thousand people, how many will vote?

3. Sam drove 320 miles in 2.5 hours, how long will it take to drive 500 at the same speed?

4. To make macaroni and cheese for 2 people the recipe calls for 6 ounces of pasta. How many ounces of pasta will I need if I need to feed 5 people?

5. The dosage of a certain medication is 2 ounces for every 50 pounds of body weight. How many ounces of the medication are required for a person who weighs 175 pounds?

6. A life insurance policy costs $6.88 for every $1,000 of insurance. At this rate, what is the cost for $17,500 worth of life insurance?

Similar triangles

Remember: two triangles are *similar* if they have the same shape but maybe different sizes. Having the same shape means that the three angles of one triangle are equal to the three corresponding angles in the other. For example, in the picture, $\angle A = \angle A'$, $\angle B = \angle B'$, and $\angle C = \angle C'$:
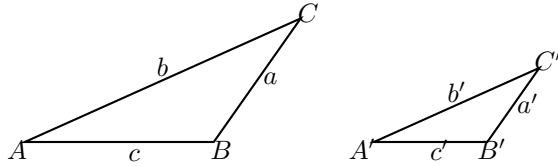
The main property of similar triangles is that their corresponding sides (the ones opposite to corresponding angles) are proportional:

$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}.$$

This means that if we know the length of all the sides of one triangle and one of the the other triangle, we can find the other two sides.
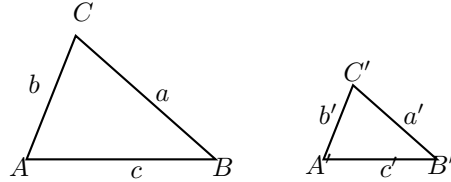
Example: In the triangles above, suppose that $a = 6$in, $c = 8$in, and $c' = 6$in; find a'.

Solution: The proportion $\dfrac{c'}{c} = \dfrac{a'}{a}$ gives $\dfrac{6}{8} = \dfrac{a'}{6}$. Therefore we have $8a' = 36$, and therefore $a' = 36/8 = 4.5$. Therefore a' is 4.5in.
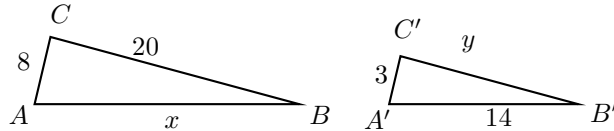
<u>Exercises</u>

7. In the triangles above, suppose that $b = 15$, $c = 10$, $a' = 9$ and $b' = 6$. Find c' and a.

8. In the triangles above, suppose that $a = 12$, $b = 20$, $c = 15$ and $a' = 7$. Find b' and c'.

Consider the following similar triangles

9. If $a = 30$ft, $b = 20$ft, $c = 40$ft and $a' = 20$ft, find b' and c'.

10. If $a' = 16$in, $b' = 10$in, $c' = 18$in and $c = 24$in, find a' and c'.

11. If $a = 14$in, $a' = 5$in, $b = 10$in and $c' = 9$in, find b' and c.

12. In the next similar triangles, find x and y.

13. In the next similar triangles, find x and y.