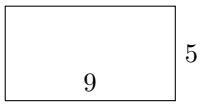
Perimeter, Area, Pythagorean Theorem

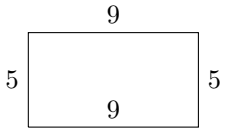
Perimeter:

Remember: A polygon is a plane figure with 3 or more straight sides. Triangles, squares, rectangles..., are all examples of polygons. The *perimeter* of a polygon is the sum of the lengths of all its sides.

Example: Find the perimeter of the following rectangle:

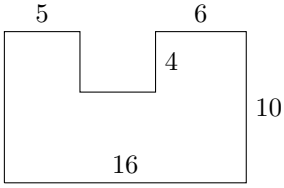


Although only two sides are given, since it is a rectangle we know that opposite sides have the same length, and therefore we can fill in the remaining sides:

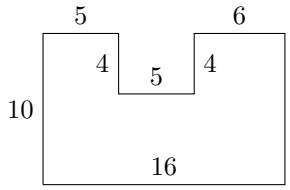


This gives that the perimeter is $5 + 9 + 5 + 9 = 28$.

Example: Find the perimeter of the following figure:



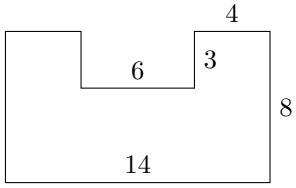
Again, some of the lengths of the sides are missing, but they can be figured out. The left side is clearly equal to the right side, so its length is 10. Also, the smaller vertical sides are equal, so they are both length 4. Finally, to find the length of the small side in the center (let us call it a), note that the total width is 16, so $5 + a + 6 = 16$, and therefore $a = 5$. So we have:



and the perimeter of the figure is $10+16+10+6+4+5+4+5 = 60$.

Exercises: Find the perimeter of the following figures.

1.



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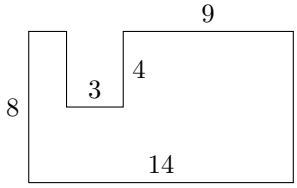

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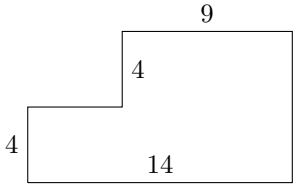

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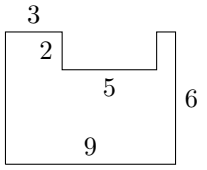

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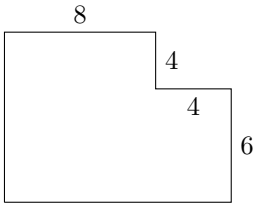

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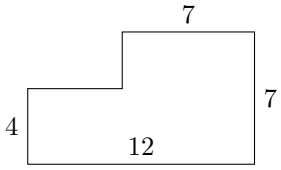

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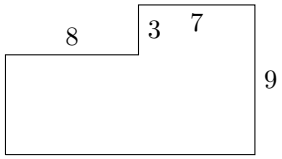

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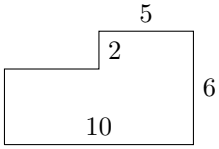

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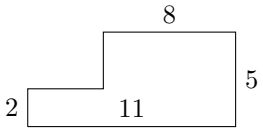
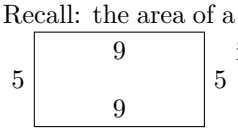


Area:

Recall: the area of a rectangle is the product of its width and its height. For example, the area of the in the rectangle

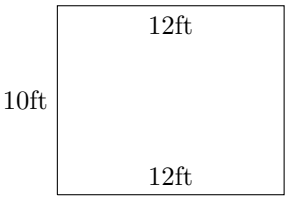


is $9 \times 5 = 45$. Area is measured in *square* units. So for example if in the previous rectangle all the lengths are given in inches, the area will be measured in *square* inches (written sq. in. or in^2).

What does area really mean? A way to understand it is as follows: imagine that you need to paint a rectangle of

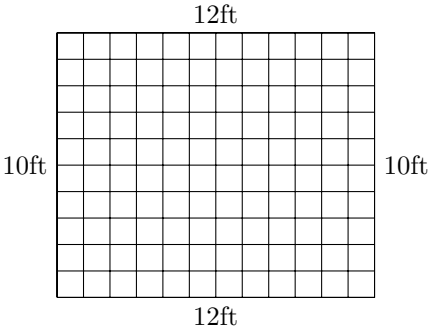
dimensions 12ft by 10ft (that's about the size of a wall in a big room). How much paint do you need to buy? You go to the shop and you are told that you to paint a square with sides one foot you need one pint of paint (this is unrealistically too much, but makes the explanation easier). In pictures:

We need 1 pint of paint to paint 

How much paint do I need to paint 

?

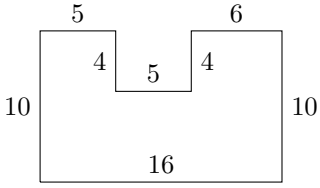
This is actually not hard: we can divide the rectangle into small 1×1 squares and then count them:

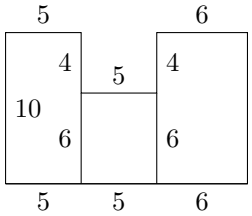


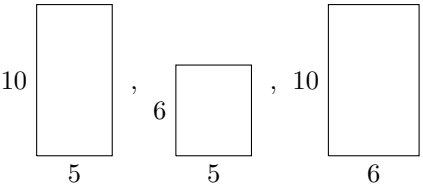
Each row has 12 squares, and there are 10 rows, so there are a total of $12 \times 10 = 120$ squares of size 1ft by 1ft. Therefore, you need 120 pints of paint to paint that wall.

This is why the area of a rectangle is defined as the product of its dimensions.

To find the area of amore complicated figure, divide it in rectangles, find the area of each rectangle and add the results up (or if you prefer, divide the figure into 1×1 squares and count how many fit in the figure). Alternatively, sometimes you can also write the area as the difference of the areas of two rectangles.

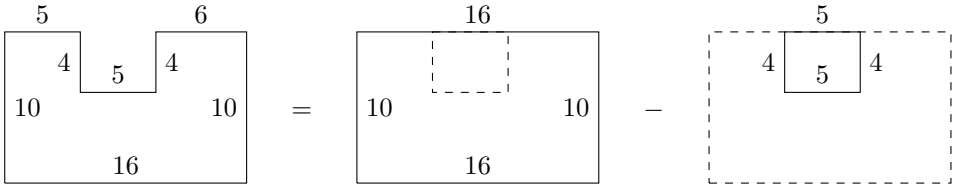
For example: find the area of (all lengths in inches): 

Divide it into smaller rectangles: 

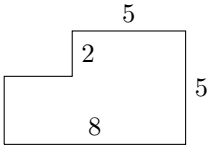
. We get 3 rectangles: 

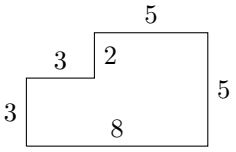
The areas of these rectangles are $10 \times 5 = 50$, $6 \times 5 = 30$, and $10 \times 6 = 60$ respectively. Therefore the area of the figure is $50 + 30 + 60 = 140$ sq. in.

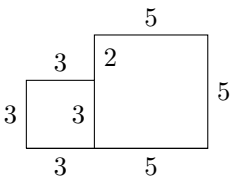
Alternatively we could have thought of the figure as a rectangle of dimensions 10 by 16 to which we have removed a little rectangle of dimensions 4 by 5 at the top:

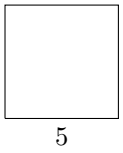


Therefore the area of the figure is the area of the first rectangle, which is $16 \times 10 = 160$, minus the area of the removed rectangle, which is $4 \times 5 = 20$. This gives $160 - 20 = 140$ sq. in., same as before.

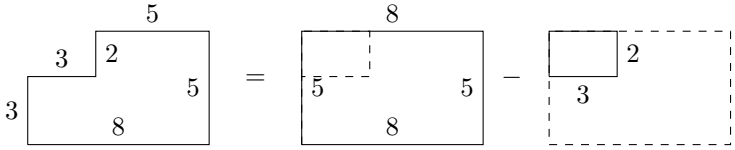
Example: Find the area of the following figure: 

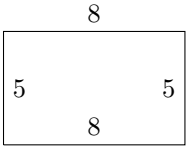
First fill in the missing lengths. Since the total width is 8, the horizontal lengths at the top have to add to 8. Also, the total height is 5, so the leftmost side has to have length 3. We get 

Method 1: divide it into smaller rectangles: 

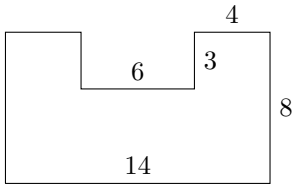
- The area of the smaller rectangle:  is $3 \times 3 = 9$.
- The area of the bigger rectangle:  is $5 \times 5 = 25$.
- Therefore the area of the figure is $25 + 9 = 34$.

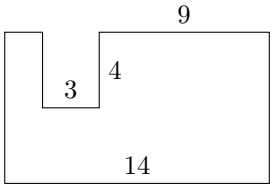
Method 2: The figure is a big rectangle with a smaller rectangle cut off:

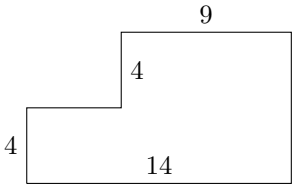


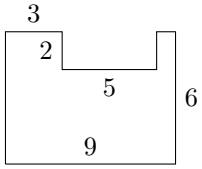
- The area of the big rectangle:  is $5 \times 8 = 40$.
- The area of the smaller rectangle:  is $2 \times 3 = 6$.
- Therefore the area of the figure is $40 - 6 = 34$.

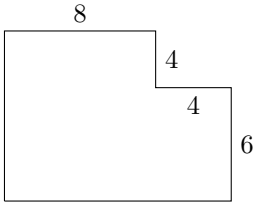
Exercises: Find the area of the following figures:

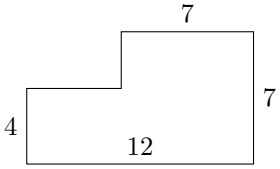
10. 

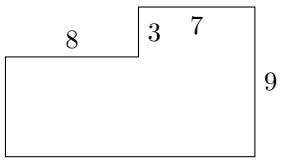
11. 

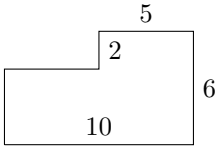
12. 

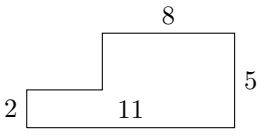
13. 

14. 

15. 

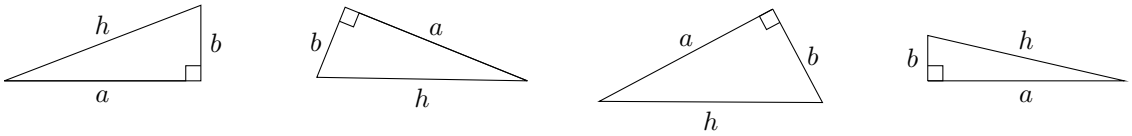
16. 

17. 

18. 

Pythagorean Theorem

An angle of 90° is called a *right* angle. Right angles are often denoted with a square arc, as in the examples below. A triangle that has a right angle is called a *right* triangle. Here are some examples:



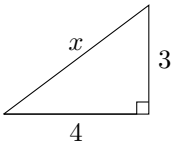
Note that the side of a right triangle that is opposite the right angle is always longer than the others. This side is called the **hypotenuse**. The other two sides are called the **legs**.

It is important to identify the hypotenuse of a right triangle - just look at the side opposite the right angle.

The pythagorean theorem says that the square of the hypotenuse is equal to the sum of the squares of the legs. In other words,

$$h^2 = a^2 + b^2.$$

Given a right triangle of which we only know the lengths of two sides, this formula lets us find the length of the other side!

For example: in the following triangle, find x: 

In this case, the hypotenuse if x. The legs are 4 and 3. Therefore we have $x^2 = 4^2 + 3^2$, which gives $x^2 = 16 + 9$, so $x^2 = 25$, and we get $x = 5$.

Example: The hypotenuse of a right triangle is 15 feet long. One of the legs is 9 feet long. What is the length of the other leg?

In this case, $h = 15$, and $a = 9$ (it does not really matter which leg we call a and which we call b). Thus we have $15^2 = 9^2 + b^2$. This gives $225 = 81 + b^2$. Subtracting 81 from both sides we get $144 = b^2$, and taking square roots of both sides we get $b = 12$. Thus the length of the other leg is 12 feet.

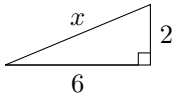
Exercises: Solve the following exercises. If the square root is not exact, leave the answer as the square root of a number without calculating it.

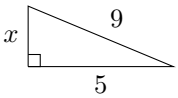
19. The hypotenuse of a right triangle is 10 feet long. One of the legs is 6 feet long. What is the length of the other leg?

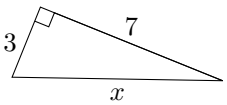
20. The legs of a right triangle are 10 feet and 9 feet long. What is the length of the hypotenuse?

21. The hypotenuse of a right triangle is 7 feet long. One of the legs is 5 feet long. What is the length of the other leg?

22. The legs of a right triangle are both 9 feet long. What is the length of the hypotenuse?

23. In the following triangle, find x: 

24. In the following triangle, find x: 

25. In the following triangle, find x: 

26. The hypotenuse of a right triangle is 15 feet long. One of the legs is 4 feet long. What is the length of the other leg?

27. The legs of a right triangle are 4 feet and 8 feet long. What is the length of the hypotenuse?

28. The hypotenuse of a right triangle is 12 feet long. One of the legs is 5 feet long. What is the length of the other leg?