Equivalent fractions. Professor Luis Fernández
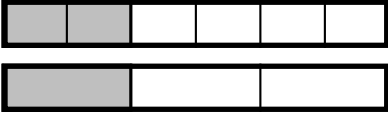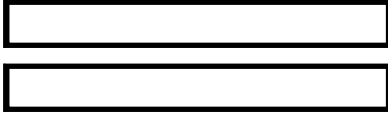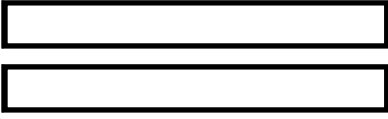
Recall: two fractions $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are equivalent if they are the same number, and then we write $\dfrac{a}{b} = \dfrac{c}{d}$.
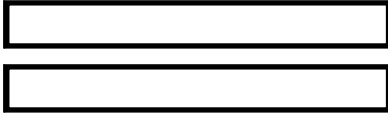
In each exercise, draw the fractions and determine if they are equivalent.
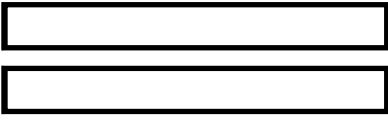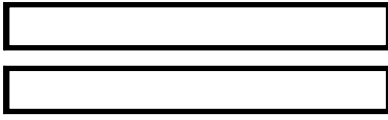
1.
$$\boxed{\quad} = \frac{2}{3}$$
$$\boxed{\quad} = \frac{1}{3}$$
Yes they are (example).

2.
$$\boxed{\quad} = \frac{2}{8}$$
$$\boxed{\quad} = \frac{1}{4}$$

3.
$$\boxed{\quad} = \frac{4}{6}$$
$$\boxed{\quad} = \frac{2}{3}$$

4.
$$\boxed{\quad} = \frac{4}{5}$$
$$\boxed{\quad} = \frac{8}{10}$$

5.
$$\boxed{\quad} = \frac{6}{9}$$
$$\boxed{\quad} = \frac{9}{12}$$

6.
$$\boxed{\quad} = \frac{2}{6}$$
$$\boxed{\quad} = \frac{3}{9}$$

Two fractions $\dfrac{a}{b} = \dfrac{c}{d}$ are equivalent if we can multiply and/or divide a and b, BOTH BY THE SAME NUMBER, to obtain c and d, respectively.

For example: $\dfrac{8}{12} = \dfrac{2}{3}$ because if we divide 8 and 12 by the same number (4) we get 2 and 3, respectively:

$8 \div 4 = 2$ and $12 \div 4 = 3$.

For example: $\dfrac{9}{12} = \dfrac{6}{8}$ because if we divide both 9 and 12 by 3, and then we multiply both by 2, we get 6 and 8, respectively:

$9 \div 3 \cdot 2 = 6$ and $12 \div 3 \cdot 2 = 8$.

For each exercise, determine if the given fractions are equivalent.

7. $\dfrac{5}{15}$ and $\dfrac{1}{3}$ $\qquad\qquad$ $\dfrac{2}{8}$ and $\dfrac{2}{4}$

8. $\dfrac{3}{4}$ and $\dfrac{12}{16}$ $\qquad\qquad$ $\dfrac{4}{6}$ and $\dfrac{10}{15}$

9. $\dfrac{3}{4}$ and $\dfrac{15}{20}$ $\qquad\qquad$ $\dfrac{3}{8}$ and $\dfrac{9}{50}$

10. $\dfrac{15}{21}$ and $\dfrac{5}{7}$ $\qquad\qquad$ $\dfrac{3}{6}$ and $\dfrac{5}{10}$

In the following exercises, write a fraction that is equivalent with the given one (note: there are several valid answers).

11. $\dfrac{15}{21} = \dfrac{5}{7}$ (example) $\qquad$ **12.** $\dfrac{2}{3} = \underline{\quad\quad}$ $\qquad$ **13.** $\dfrac{7}{2} = \underline{\quad\quad}$

14. $\dfrac{18}{12} = \underline{\quad\quad}$ $\qquad$ **15.** $\dfrac{14}{7} = \underline{\quad\quad}$ $\qquad$ **16.** $\dfrac{4}{5} = \underline{\quad\quad}$

17. $\dfrac{6}{8} = \underline{\quad\quad}$ $\qquad$ **18.** $\dfrac{3}{9} = \underline{\quad\quad}$ $\qquad$ **19.** $\dfrac{8}{16} = \underline{\quad\quad}$

Recall: simplifying a fraction means to find an equivalent fraction whose numerator and denominator are smaller than the original.

To simplify a fraction, divide BOTH its numerator and denominator by the SAME number, if possible. Keep doing this until there are no common factors of the numerator and the denominator.

When one cannot simplify any further, we say that the fraction is *in lowest terms*.

In each of the following exercises, write the given fraction in lowest terms.

20. $\dfrac{20}{12} = \underline{\qquad}$

21. $\dfrac{8}{6} = \underline{\qquad}$

22. $\dfrac{15}{10} = \underline{\qquad}$

23. $\dfrac{2}{8} = \underline{\qquad}$

24. $\dfrac{10}{3} = \underline{\qquad}$

25. $\dfrac{13}{6} = \underline{\qquad}$

26. $\dfrac{20}{15} = \underline{\qquad}$

27. $\dfrac{14}{16} = \underline{\qquad}$

28. $\dfrac{18}{12} = \underline{\qquad}$

29. $\dfrac{20}{60} = \underline{\qquad}$

30. $\dfrac{35}{15} = \underline{\qquad}$

31. $\dfrac{18}{48} = \underline{\qquad}$

32. $\dfrac{50}{20} = \underline{\qquad}$

33. $\dfrac{42}{32} = \underline{\qquad}$

34. $\dfrac{36}{64} = \underline{\qquad}$

It is often necessary to change a fraction to an equivalent one that has a prescribed denominator denominator. For example if I want to write a fraction equivalent to $\dfrac{3}{4}$ but with 20 as denominator, I would need to multiply 3 and 4 by a suitable number so that the new denominator is 20 (so the multiplier is 5, because $4 \times 5 = 20$):

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}.$$

Example: to find the numerator of the second fraction so that $\dfrac{5}{3} = \dfrac{}{12}$, we need to multiply BOTH 5 and 3 by some number a so that $3 \times a = 12$. It is clear that $a = 4$, so we have

$$\frac{5}{3} = \frac{5 \times 4}{3 \times 4} = \frac{20}{12}.$$

In each exercise, find the appropriate numerator so that the two fractions are equivalent.

35. $\dfrac{2}{5} = \dfrac{\mathbf{4}}{10} \quad \left(\dfrac{2 \cdot 2}{5 \cdot 2} = \dfrac{4}{10}\right)$ (ex.)

36. $\dfrac{3}{4} = \dfrac{}{12}$

37. $\dfrac{5}{2} = \dfrac{}{8}$

38. $\dfrac{5}{3} = \dfrac{}{9}$

39. $\dfrac{4}{6} = \dfrac{}{18}$

40. $\dfrac{5}{10} = \dfrac{}{30}$

41. $\dfrac{3}{5} = \dfrac{}{30}$

42. $\dfrac{2}{7} = \dfrac{}{35}$

43. $\dfrac{3}{8} = \dfrac{}{40}$

44. $\dfrac{7}{5} = \dfrac{}{25}$

45. $\dfrac{3}{11} = \dfrac{}{11}$

46. $\dfrac{5}{6} = \dfrac{}{6}$