Evaluating expressions

Remember: A mathematical expression is a combinations of numbers, letters, and operation symbols that make sense when the letters are substituted by numbers. The letters are normally called *variables* because they represent numbers that can vary.

Examples: $\dfrac{x+y}{2x-1}$, x^2+4, $T-2H$, and $\sqrt{b^2-4ac}$ are all mathematical expressions.

To evaluate a mathematical expression, simply substitute the value of the variable or variables into the expression and simplify the result.

IMPORTANT: when you substitute variables by a number, always write the number **in parenthesis** when you substitute. This way you will avoid confusions and errors.

For example: Evaluate $\dfrac{x+y}{x^2+4}$ when $x=-1$, $y=2$:

- Substitute each variable by its given value, written in parenthesis: $\dfrac{(-1)+(2)}{(-1)^2+4}$.
- Simplify the expression, remembering to use order of operations appropriately: $\dfrac{(-1)+(2)}{(-1)^2+4}=\dfrac{1}{1+4}=\dfrac{1}{5}$.

NOTE: It is always useful to write a dot "$\cdot$" at those places where we have to multiply but there is no multiplication symbol (just an empty space). So for example, if you have the expression $3mn$, start by writing it as $3\cdot m\cdot n$. It makes things more clear.

Exercises:

Evaluate the following expressions

1. $a+6b$ if $a=4$, $b=8$

2. $a+6b$ if $a=-7$, $b=-2$

3. $4xy$ if $x=4$, $b=-3$

4. x^2-y^2+3 if $x=2$, $y=3$

5. x^2-y^2+3 if $x=-2$, $y=1$

6. x^2-y^2+3 if $x=-1$, $y=-3$

7. $\sqrt{x^2+y^2}$ if $x=3$, $y=4$

8. $\sqrt{x^2+y^2}$ if $x=-5$ $y=12$

9. $\dfrac{t+3s}{4st}$ if $s=4$, $t=-3$

10. $\dfrac{t+3s}{4st}$ if $s=-3$, $t=-1$

11. $\dfrac{4d+e}{2d-3e}$ if $d=-2$ $e=3$

12. $\dfrac{9}{5}C+32$ if $C=-10$.

13. $-b+\sqrt{b^2-4ac}$ if $a=1$, $b=2$, $c=-3$

14. $-b+\sqrt{b^2-4ac}$ if $a=2$, $b=-5$, $c=-3$

15. $-b-\sqrt{b^2-4ac}$ if $a=1$, $b=-6$, $c=8$

16. $-b-\sqrt{b^2-4ac}$ if $a=3$, $b=4$, $c=-4$

Formulas

A formula expresses one quantity in terms of other quantities. For example, the area of a rectangle is:

$$\text{Area} = \text{length} \times \text{width}.$$

Formulas give important quantities to be used in real life problems (for example, if you want to paint a room, you need to estimate the area to be painted in order to buy the right amount of paint). To evaluate a formula, just substitute the given value of the variables as before.

Example: Find the area of a rectangle with width 12ft and height 5 ft: Since we know that the area of a rectangle is Area = length $\times$ width, we substitute: Area $=(12)\times(5)=60$. Area is written in square feet, so the answer is 60 square feet.

Example: To convert from degrees Celsius to degrees Fahrenheit the formula is $F=\dfrac{9}{5}C+32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius. Let us find the temperature in Fahrenheit when $C=100$. Substitute: $F=\frac{9}{5}(100)+32=\frac{900}{5}+32=180+32=112$ degrees Fahrenheit

Exercises:

Evaluate the following formulas for the given values of the variables.

17. $F=\dfrac{9}{5}C+32$ when $C=25$ degrees.

18. $F=\dfrac{9}{5}C+32$ when $C=97$ degrees.

19. $C=\dfrac{5}{9}(F-32)$ when $F=81$ degrees.

20. $C=\dfrac{5}{9}(F-32)$ when $F=32$ degrees.

21. The child's dosage for a medicine is given by the formula $C = \dfrac{t+1}{t+15}\,A$, where A is the adult dosage and t is the age of the child. Find the dosage for a 9 year old if the adult dosage is 36 miligrams.

22. Same as the previous problem for a 12 year old if the adult dosage is 30 miligrams.

23. The formula $P = D(1+r)^t$ gives the amount of money in an investment after t years when the initial invested amount is D dollars and the interest rate is r (r written as a decimal). Find P after 2 years when the initial investment was \$1000, at an interest rate of 12%.

24. Using the same formula as in the previous problem, find P after 10 years when the initial investment was \$2000, at an interest rate of 15% (you can use a calculator for this one).

25. Using the same formula as in the previous problem, find P after 30 years when the initial investment was \$2000, at an interest rate of 15% (you can use a calculator for this one).

26. The area of a triangle is given by the formula $A = \dfrac{\text{base} \times \text{height}}{2}$. Find the area of a triangle with base 5 inches and height 4 inches.

27. The area of a triangle is given by the formula $A = \dfrac{\text{base} \times \text{height}}{2}$. Find the area of a triangle with base 12 inches and height 7 inches.

28. The perimeter of a rectangle is the sum of the lengths of its four sides. Find the perimeter of a rectangle of length 23 and height 10.

29. The perimeter of a rectangle is the sum of the lengths of its four sides. Find the perimeter of a rectangle of length 45 and height 60.

Function notation

A common way to express a formula in mathematics is *function notation*. As in the formulas before, it expresses a quantity (say y) in terms of other quantity or quantities (say x), and we say that y is a function of x. We write this in the form

$$y = f(x),$$

and read it as "y equals f of x". The letter f in this case is just the name of the function, and we could have used any other name. For example, we can write the formula for degree conversion above as $F(C) = \frac{9}{5}C + 32$. The only real difference from before is that the name of the variable (C in this case) is written after the dependent quantity in parenthesis.

This way of writing things is easier and shorter: instead of writing as before, for example, "find F when $C = 15$", we now can write "find $F(15)$".

Example: If $g(x) = 3x + 5$, find $g(2)$. Answer: $g(2) = 3 \cdot (2) + 5 = 6 + 5 = 11$.

Example: If $\mathrm{SUM}(x, y) = x + y$, find $\mathrm{SUM}(4, 2)$. Answer: $\mathrm{SUM}(4, 2) = (4) + (2) = 6$.

<u>Exercises</u>:

30. If $f(x) = x^2 + 1$, find $f(3)$. **31.** If $f(x) = x^2 + 1$, find $f(-5)$. **32.** If $h(t) = (t-3)(t+4)$, find $h(5)$.

For the function $g(x) = x^2 - 4x + 3$, find

33. $g(4)$ $g(-3)$ $g(6)$ $g(-4)$

For the function $r(t) = 4t - 6$, find

34. $r(3)$ $r(-5)$ $r(-9)$ $r(-4)$

For the function $T(a) = \dfrac{a+3}{a+2}$, find

35. $T(2)$ $T(-1)$ $T(1)$ $T(-4)$