

Multiplication by negative powers of 10

Remember: $10^{-k} = \frac{1}{10^k} = 0.00\dots 01$ (k decimals total). For example, $10^{-3} = 1/1000 = 0.001$.

Therefore, multiplying by 10^{-k} is the same as dividing by 10^k , which amounts to *moving the decimal point k places to the left*.

Examples: $157 \times 10^{-2} = 1.57$. $134.456 \times 10^{-3} = 0.134456$. $0.23 \times 10^{-5} = 0.0000023$.

Exercises: Multiply

- | | | |
|-------------------------------|------------------------------|-----------------------------|
| 1. $445 \times 10^{-2} =$ | 2. $42965 \times 10^{-3} =$ | 3. $153.5 \times 10^{-3} =$ |
| 4. $345.654 \times 10^{-5} =$ | 5. $179.65 \times 10^{-4} =$ | 6. $0.05 \times 10^{-4} =$ |
| 7. $215.34 \times 10^{-1} =$ | 8. $423653 \times 10^{-7} =$ | 9. $0.003 \times 10^{-1} =$ |

Multiplication of powers of 10 by powers of 10

Recall: To multiply by 10^k , move the decimal point k spaces to the right. To multiply by 10^{-k} , move the decimal point k spaces to the left.

This implies: $10^n \times 10^k = 10^{n+k}$ (it does not matter whether k or n are positive or negative—it always works).

Examples: $10^3 \times 10^4 = 10000000 = 10^7$. $10^{-3} \times 10^4 = 10^{-3} \times 10000 = 10.000 = 10^1$.

Exercises: Multiply and write the result as a power of 10.

- | | | |
|--------------------------------|--------------------------------|--------------------------------|
| 10. $10^4 \times 10^2 =$ | 11. $10^7 \times 10^{-3} =$ | 12. $10^{-4} \times 10^6 =$ |
| 13. $10^{-5} \times 10^3 =$ | 14. $10^{-3} \times 10^{-4} =$ | 15. $10^{-7} \times 10^{-4} =$ |
| 16. $10^{12} \times 10^{11} =$ | 17. $10^{-23} \times 10^7 =$ | 18. $10^k \times 10^n =$ |

Division of powers of 10 by powers of 10

Recall: To divide by 10^k , move the decimal point k spaces to the left. To divide by 10^{-k} , move the decimal point k spaces to the right.

This implies: $\frac{10^n}{10^k} = 10^{n-k}$ (it does not matter whether k or n are positive or negative—it always works).

Examples: $\frac{10^3}{10^4} = \frac{1000}{10000} = 0.1 = 10^{-1}$. $\frac{10^{-3}}{10^4} = \frac{0.001}{10000} = 0.0000001 = 10^{-7}$. $\frac{10^4}{10^{-2}} = \frac{10000}{0.01} = 1000000 = 10^6$.

Exercises: Divide and write the result as a power of 10.

- | | | |
|---------------------------------|---------------------------------|---------------------------------|
| 19. $\frac{10^4}{10^2} =$ | 20. $\frac{10^7}{10^{-3}} =$ | 21. $\frac{10^{-4}}{10^6} =$ |
| 22. $\frac{10^{-5}}{10^3} =$ | 23. $\frac{10^{-3}}{10^{-4}} =$ | 24. $\frac{10^{-7}}{10^{-4}} =$ |
| 25. $\frac{10^{12}}{10^{11}} =$ | 26. $\frac{10^{-23}}{10^7} =$ | 27. $\frac{10^k}{10^n} =$ |

Scientific Notation

Remember: A number is in scientific notation if it has exactly one digit in the whole part, and this digit is not zero.

Examples: 23.45 is not in scientific notation because it has two digits (2 and 3) in the whole part.

4.45004 is in scientific notation because the whole number part consists only of a nonzero digit.

0.45 (or .45) is not in scientific notation because the whole number part consist only of a 0.

Exercises: Which of these numbers are written in scientific notation?

- | | | |
|------------|-----------|-----------|
| 28. 0.23 | 29. 2.55 | 30. 45.34 |
| 31. 123.23 | 32. 24.55 | 33. 0.004 |

Exercises: The following numbers are written in scientific notation. Write them in usual notation.

- | | | |
|--|-----------------------------|------------------------------|
| 34. $2.3 \times 10^3 = 2300$ (example) | 35. $2.55 \times 10^5 =$ | 36. $4.534 \times 10^{-4} =$ |
| 37. $2.3 \times 10^{-3} =$ | 38. $1.23 \times 10^4 =$ | 39. $8.004 \times 10^{-4} =$ |
| 40. $1.76 \times 10^8 =$ | 41. $7.25 \times 10^{-6} =$ | 42. $2.418 \times 10^{-1} =$ |

Remember: To write a number in scientific notation,

- Move the decimal place to the right or to the left until only one nonzero digit appears in the whole part (before the decimal point).
- Count how many places you moved the decimal point (say k places) and multiply the number by 10^k if you moved the point to the left and by 10^{-k} if you moved the point to the right.

NOTE: To remember which sign of the exponent you need to use, think that when you moved the decimal point,

- If you made the number smaller, you need to multiply by a positive power of 10 to compensate.
- If you made the number greater, you need to multiply by a negative power of 10 to compensate.

Examples: $465.13 = 4.6513 \times 10^2$ (positive power because 4.6513 is smaller than 465.13).

Exercises: Write the following numbers in scientific notation.

43. $23.3 =$ (example)

44. $20355 =$

45. $534.44 =$

46. $0.00003 =$

47. $1.45355 =$

48. $5449994.34 =$

49. $0.001001 =$

50. $255443.234 =$

51. $5000000 =$

52. $24.54 \times 10^4 =$

53. $322 \cdot 10^{-3} =$

54. $534.44 \times 10^{34} =$

Multiplication and division in scientific Notation

Remember: To multiply two numbers that are written in scientific notation,

1. Multiply the number parts and the powers of 10 parts separately. Remember that to multiply the powers of 10 you only need to add the exponents.
2. Write the result of multiplying the number parts in scientific notation (if it is not in scientific notation already).
3. If you get an extra power of 10 from the previous item, multiply it with the other powers of 10.

Example: $(3 \times 10^4) \cdot (2 \times 10^3) = (3 \times 2) \times (10^4 \times 10^3) = 6 \times 10^7$.

Example: $(3.1 \times 10^{-3}) \cdot (5 \times 10^6) = (3.1 \times 5) \times (10^{-3} \times 10^6) = 15.5 \times 10^{-3+6} = 15.5 \times 10^3 = 1.55 \times 10^4$.

NOTE: It is easy to forget to write the final result in scientific notation. Make sure you do step 2.

Exercises: Multiply and write the product in scientific notation.

55. $(2.3 \times 10^3) \cdot (3 \times 10^6) =$

56. $(4.5 \times 10^5) \cdot (2.1 \times 10^{12}) =$

57. $(6.3 \times 10^{-4}) \cdot (5 \times 10^{13}) =$

58. $(9 \times 10^{-7}) \cdot (3 \times 10^{-12}) =$

59. $(2.4 \times 10^8) \cdot (4.2 \times 10^{-10}) =$

60. $(3.3 \times 10^{-8}) \cdot (5.3 \times 10^2) =$

61. $(5.7 \times 10^6) \cdot (3.1 \times 10^{-2}) =$

62. $(4.5 \times 10^{-6}) \cdot (6.1 \times 10^{11}) =$

63. $(1.11 \times 10^{-4}) \cdot (5.11 \times 10^{10}) =$

Remember: To divide two numbers that are written in scientific notation,

1. Divide the number parts and the powers of 10 parts separately. Remember that to divide the powers of 10 you only need to subtract the exponents of the denominator from the exponent in the numerator.
2. Write the result of dividing the number parts in scientific notation (if it is not in scientific notation already).
3. If you get an extra power of 10 from the previous item, multiply it with the other powers of 10.

Example: $\frac{3 \times 10^4}{2 \times 10^3} = \frac{3}{2} \times \frac{10^4}{10^3} = 1.5 \times 10^{4-3} = 1.5 \times 10^1$.

Example: $\frac{3.1 \times 10^{-3}}{5 \times 10^6} = \frac{3.1}{5} \times \frac{10^{-3}}{10^6} = 0.62 \times 10^{-3-6} = 0.62 \times 10^{-9} = 6.2 \times 10^{-1} \times 10^{-9} = 6.2 \times 10^{(-1)+(-9)} = 6.2 \times 10^{-10}$.

Exercises: Divide and write the product in scientific notation.

64. $\frac{4.5 \times 10^3}{3 \times 10^2} =$

65. $\frac{1.4 \times 10^5}{5 \times 10^2} =$

66. $\frac{6.3 \times 10^{-4}}{3 \times 10^{13}} =$

67. $\frac{3 \times 10^{-7}}{6 \times 10^{-12}} =$

68. $\frac{4.2 \times 10^8}{2.4 \times 10^{-10}} =$

69. $\frac{3.6 \times 10^{-8}}{4.5 \times 10^2} =$