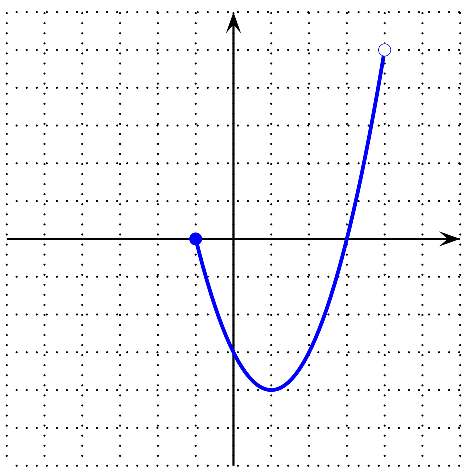
BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

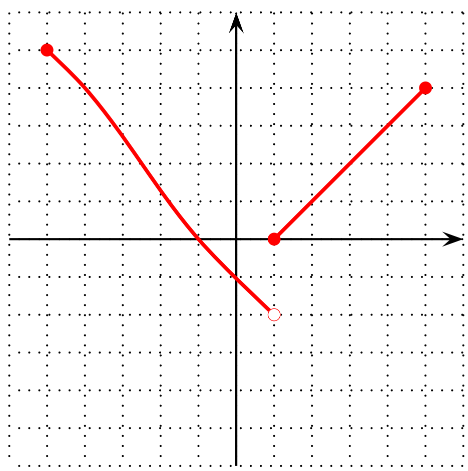
MTH30 Review Sheet

1. Given the functions f and g described by the graphs below:

$y = f(x)$



$y = g(x)$



(a) Find:

i. The domain of f

ii. The range of f

iii. An open interval on which f is increasing

iv. An open interval on which f is decreasing

v. the local minimum of f

vi. Write the set of all x where $f(x)$ is negative in interval notation.

vii. Write the set of all x where $f(x)$ is positive in interval notation.

(b) Find:

i. The domain of g

ii. The range of g

iii. The x-intercept of g

iv. The y-intercept of g

v. An interval on which g is one-to-one

(c) Evaluate the following, if they exist:

i. $g(1)$

ii. $(f+g)(1)$

iii. $(f-g)(4)$

iv. $\left(\dfrac{g}{f}\right)(-1)$

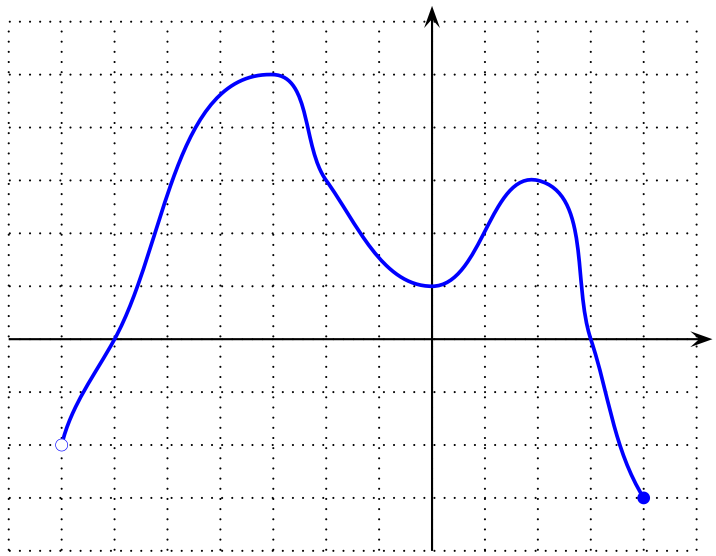
v. $(f \circ g)(1)$

vi. $(g \circ g)(-5)$

vii. $(f \circ f)(3)$

2. Let $f(x) = \sqrt{x^2+4x+4}$ and $g(x) = \dfrac{x^2-1}{\sqrt{1-x}}$.
   (a) Find the domains of f and g. Give your answer using interval notation.
   (b) Evaluate, if defined: $f(g(0))$; $g(f(0))$; $(f \cdot g)(0)$

3. Given the graph of $y = f(x)$, answer the following questions.



   (a) Find the domain of f
   (b) Find the range of f
   (c) Over which open intervals is f increasing?
   (d) Over which open intervals is f decreasing?
   (e) Find $f(-3)$ and $f(4)$
   (f) Find all solutions to the equation $f(x) = 3$
   (g) Find the zeros of the function.
   (h) Does f have an inverse function? Explain.

4. For each of the functions f given below:
   A. $f(x) = \dfrac{x}{x+1}$ B. $f(x) = e^{2x-1}$ C. $f(x) = \log_2(3-x)$
   (a) Find the inverse function f^{-1}.
   (b) Verify that $f\left(f^{-1}(x)\right) = f^{-1}(f(x)) = x$
   (c) Sketch a graph of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of coordinates.

5. Consider the functions: $f(x) = e^{x^2}$ and $g(x) = \sqrt{\ln x}$. Are f and g a pair of inverse functions? Justify your answer.

6. For each pair of functions f and g given below find $f \circ g$ and $g \circ f$.
   (a) $f(x) = 2x^2 - 3x + 5$; $g(x) = 5 - 2x$.
   (b) $f(x) = \dfrac{2x}{x-5}$; $g(x) = \dfrac{5x}{x-2}$
   (c) $f(x) = x^2 - 4$; $g(x) = \sqrt{x+5}$

7. Sketch the graphs of the following linear equations:

   (a) $2x - 3y = 6$ (b) $x + 4y = 8$ (c) $y = -\frac{1}{2}x + 4$ (d) $y = 2x - 3$

8. Find the slope of the lines described by the following information:
   (a) With equation $y = \frac{2}{3}x + 4$
   (b) With equation $2x - 3y = 8$
   (c) Passing through the points $(4, -2)$ and $(5, 1)$
   (d) Perpendicular to the line with equation $x - 4y = 1$

9. Write an equation of the line described by the following information:
   (a) With slope $-\frac{1}{2}$ and passing through the point $(3, -2)$
   (b) Passing through the points $(2, -1)$ and $(-4, -3)$
   (c) perpendicular to the line with equation $y = 3x - 4$ and passing through $(1, 9)$.
   (d) Parallel to the line with equation $3x - 5y = 4$ and having the same y–intercept as the line with equation $x - 4y - 8 = 0$.

10. Solve the systems:

    (a) $\begin{cases} x + y &= 1 \\ 2x - y &= 8 \end{cases}$ (b) $\begin{cases} 5x - 2y &= 10 \\ 2x - 7y &= 14 \end{cases}$ (c) $\begin{cases} 2x + y &= 4 \\ 2x - 3y &= 1 \end{cases}$

11. For each of the the following quadratic functions $f(x)$:
    A. $f(x) = (x - 2)^2 - 1$ B. $f(x) = x^2 + 2x - 3$ C. $f(x) = -3x^2 - 6x - 4$
    (a) Find the vertex.
    (b) State the domain of f.
    (c) State the range of f.
    (d) Find the x-intercept(s).
    (e) Find the y-intercept(s).
    (f) Write the set of all x where $f(x)$ is negative in interval notation.
    (g) Write the set of all x where $f(x)$ is positive in interval notation.
    (h) Sketch the graph of $y = f(x)$.

12. The graph of a parabola $y = f(x)$ has axis of symmetry $x = -1$, vertex $(-1, 5)$, and $f(0) = 3$.
    (a) Write the equation of the parabola in standard form.
    (b) State the domain and the range of f.
    (c) Sketch a graph of $y = f(x)$.

13. For each of the the following polynomials $p(x)$:
    A. $p(x) = x^3 - 3x^2 + 4$ B. $p(x) = -x^3 + 4x^2 - x - 6$ C. $p(x) = 2x^4 + 7x^3 + 6x^2 - x - 2$
    (a) List all possible rational roots of $p(x)$, according to the Rational Zeros Theorem.
    (b) Factor $p(x)$ completely.
    (c) Find all roots of the equation $p(x) = 0$.
    (d) Determine the end behavior of the graph of $y = p(x)$.
    (e) Determine the y–intercept of the graph of $y = p(x)$

(f) Determine the x–intercepts of the graph $y = p(x)$

(g) Determine the local behavior of $y = p(x)$ near the x-intercepts.

(h) Use the above information to sketch a graph of $y = p(x)$.

14. Find the remainder of the division of $x^{122} - 20x^{51} + 60x^{34} + 1$ when divided by $x - 1$.

15. For each of the following rational functions f

A. $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - x - 2}$ B. $f(x) = \dfrac{x^2 + 2x - 3}{x^2 - 2x - 3}$ C. $f(x) = \dfrac{x^2 - 9}{x^2 - x - 2}$ D. $f(x) = \dfrac{2 - x}{x^2 + x - 2}$

E. $f(x) = \dfrac{x^2}{x^2 + 1}$

(a) Factor numerator and denominator and simplify if possible.

(b) Find the x and y intercepts of the graph of $y = f(x)$ if they exist.

(c) Find any vertical or horizontal asymptotes.

(d) Determine how the sign of $f(x)$ changes.

(e) Use the above information to sketch a graph of $y = f(x)$.

16. For each of the following rational functions f

A. $f(x) = 2^x + 1$ B. $f(x) = 3^{x+2} - 4$ C. $f(x) = log_3(x) - 2$ D. $f(x) = log_2(x + 1) - 1$

(a) Find the domain.

(b) Find the range.

(c) Find any vertical or horizontal asymptotes.

(d) Use the above information to sketch a graph of $y = f(x)$.

17. Solve the inequality and express the answer using interval notation.

A. $|2x - 1| > 3$ B. $4|2 - 3x| + 2 \leq 6$

18. Evaluate the following expressions. Give exact values whenever possible:

(a) $\log_2 \dfrac{1}{64}$

(b) $\log_9 \dfrac{\sqrt{3}}{3}$

(c) $\log_b x^3 y$, given that $\log_b x = 2$ and $\log_b y = 36$

(d) e^{x-y} given that $e^x = 3$ and $e^y = 4$

(e) $\log_a \left(\dfrac{x}{y}\right)$ given that $\log_a(x) = 12$ and $\log_a(y) = 4$

(f) $\ln e^{\sqrt{2}}$

(g) $\log 1000$

(h) $\log_7 31$, rounded to the nearest hundredth

(i) $\sin \dfrac{5\pi}{4}$

(j) $\tan \left(-\dfrac{7\pi}{6}\right)$

(k) $\cos \dfrac{13\pi}{6}$

(l) $\sec \dfrac{8\pi}{3}$

(m) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

(n) $\cos^{-1}\left(-\frac{1}{2}\right)$

(o) $\sin^{-1}\left(\sin\frac{\pi}{6}\right)$

(p) $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$

(q) $\cos\left(\sin^{-1}(-1)\right)$

(r) $\sin(a+b)$, if $\sin a = \frac{1}{3}$, $\cos b = \frac{3}{5}$ and $0 < a, b < \frac{\pi}{2}$

19. Find θ if

(a) $\cos\theta = \frac{\sqrt{3}}{2}$, and $\frac{3\pi}{2} < \theta < 2\pi$.

(b) $\sin\theta = -\frac{1}{2}$, and $\pi < \theta < \frac{3\pi}{2}$.

(c) $\sin\theta = \frac{\sqrt{2}}{2}$, and $\frac{\pi}{2} < \theta < \pi$

20. Solve the following equations:

(a) $\log_2 x - \log_2(x-1) = 1$

(b) $7^{x+2} = 49$

(c) $\sin^2 x = \frac{3}{4}$, where x is in the interval $[0, 2\pi)$

(d) $2\cos^2 x + 3\cos x + 1 = 0$, where x is in the interval $[0, 2\pi)$

21. Verify the following identities:

(a) $\tan^2 x + 1 = \sec^2 x$

(b) $\csc x - \sin x = \cot x \cos x$

(c) $\csc x - \cos x \cot x = \sin x$

(d) $\cos^2 x - \sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$

22. For each of the following functions

A. $f(x) = -\sin(4x - \pi)$ B. $f(x) = -3\cos(2x + \pi)$

C. $f(x) = 2\sin(3x - \frac{\pi}{2})$ D. $f(x) = \frac{1}{2}\cos\left(\frac{x}{2} - \frac{\pi}{2}\right)$

(a) Find the period of this function.

(b) Find the amplitude of the graph of $y = f(x)$

(c) Find the phase shift of the graph of $y = f(x)$

(d) Sketch two complete cycles of the graph of $y = f(x)$

23. x is in the standard position. Find $\tan x$ if $\cos x = \frac{3}{4}$, and the terminal side of x is in the quadrant IV.

The answers

1. (a) **i**. $[-1,4)$, **ii**. $[-4,5)$, **iii**. $(1,4)$, **iv**. $(-1,1)$, **v**. 1, **vi**. $(-1,3)$, **vii**. $(3,4)$

   (b) **i**. $[-5,5]$, **ii**. $(-2,5]$, **iii**. -1 and 1, **iv**. -1, **v**. $[-5,1)$ or $[1,5]$

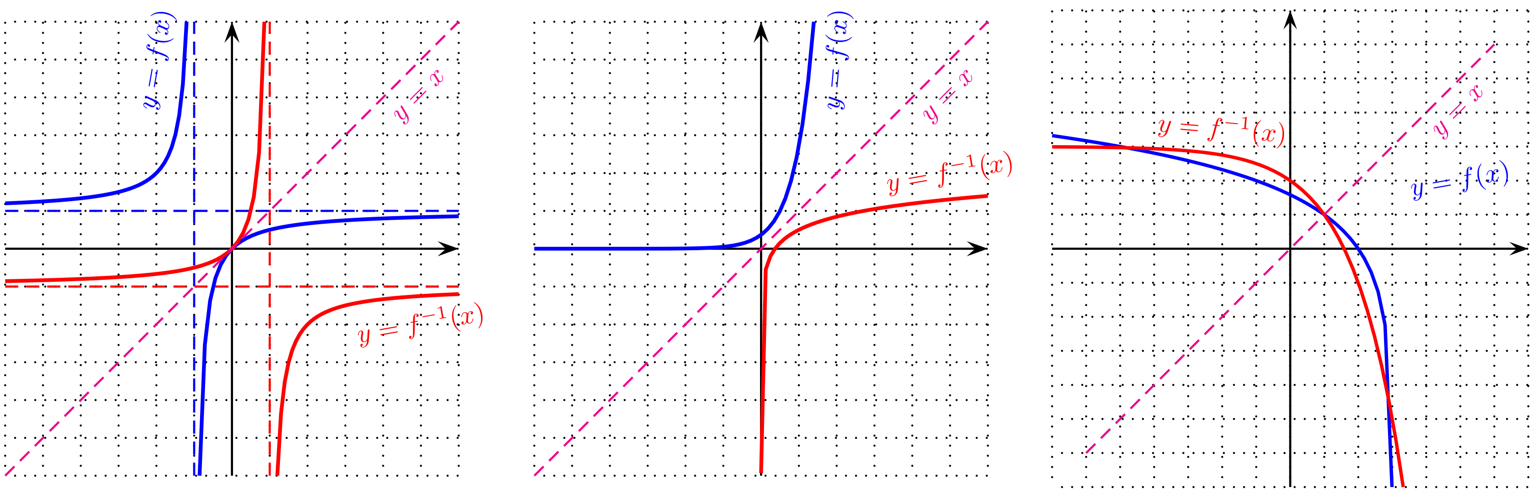
   (c) **i**. 0, **ii**. -4, **iii**. undefined, **iv**. undefined, **v**. -3, **vi** 4, **vii** -3

2. (a) Domain of f is $(-\infty,\infty)$, domain of g is $(-\infty,1)$

   (b) $f\left(g(0)\right)=1$; $g\left(f(0)\right)$ is undefined; $(f\cdot g)(0)=-2$

3. A. $(-7,4]$ B. $[-3,5]$ C. $(-7,-3)$ and $(0,2)$ D. $(-3,0)$ and $(2,4)$ E. $f(-3)=5$; $f(4)=-3$ F. $\{-5,-2,2\}$ G. -6, 3 H. No. It is not one-to-one.

4. For the graphs see Figure 1. A. $f^{-1}(x)=\dfrac{x}{1-x}$ B. $f^{1}(x)=\dfrac{\ln x+1}{2}$ C. $f^{-1}(x)=3-2^{x}$



(a) (b) (c)

Figure 1: The graphs of Question 4

5. They are not a pair of inverse functions: f is not one-to-one and thus it doesn't have an inverse function.

6. A. $(f\circ g)(x)=8x^2-34x+40$; $(g\circ f)(x)=-4x^2+6x-5$ B. $(f\circ g)(x)=x$; $(g\circ f)(x)=x$ C. $(f\circ g)(x)=x+1$; $(g\circ f)(x)=\sqrt{x^2+1}$

7. For the graphs see Figure 2.

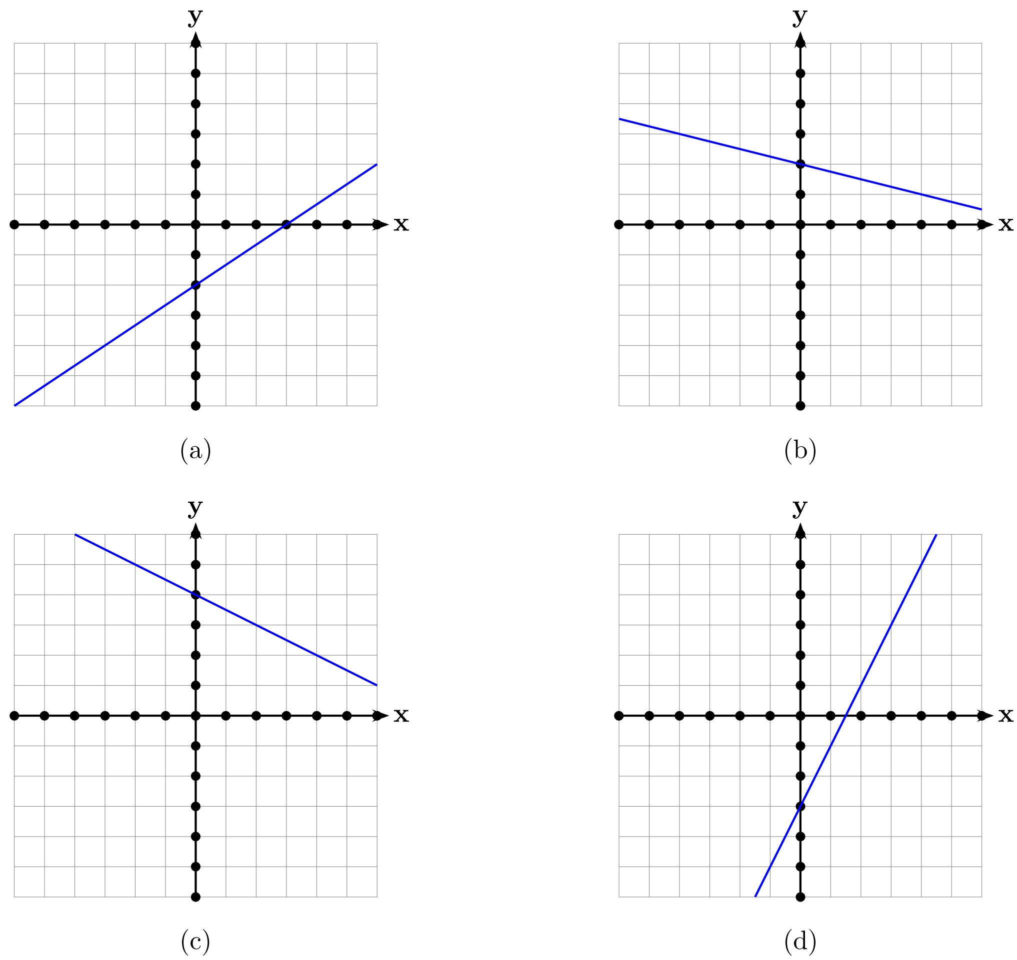


Figure 2: The graphs of Question 7

8. A. $\frac{2}{3}$ B. $\frac{2}{3}$ C. 3 D. -4

9. A. $x + 2y = -1$ B. $x - 3y = 5$ C. $y = -\frac{1}{3}x + \frac{28}{3}$ D. $y = \frac{3}{5}x - 2$

10. A. $(3, -2)$ B. $\left(\frac{42}{31}, -\frac{50}{31}\right)$ C. $\left(\frac{13}{8}, \frac{3}{4}\right)$

11. (a) A. $(2, -1)$ B. $(-\infty, \infty)$ C. $[-1, \infty)$
    (b) A. $(-1, -4)$ B. $(-\infty, \infty)$ C. $[-4, \infty)$
    (c) A. $(-1, -1)$ B. $(-\infty, \infty)$ C. $(-\infty, -1]$
    Answers to the remaining parts can be read from the graphs in Figure 3

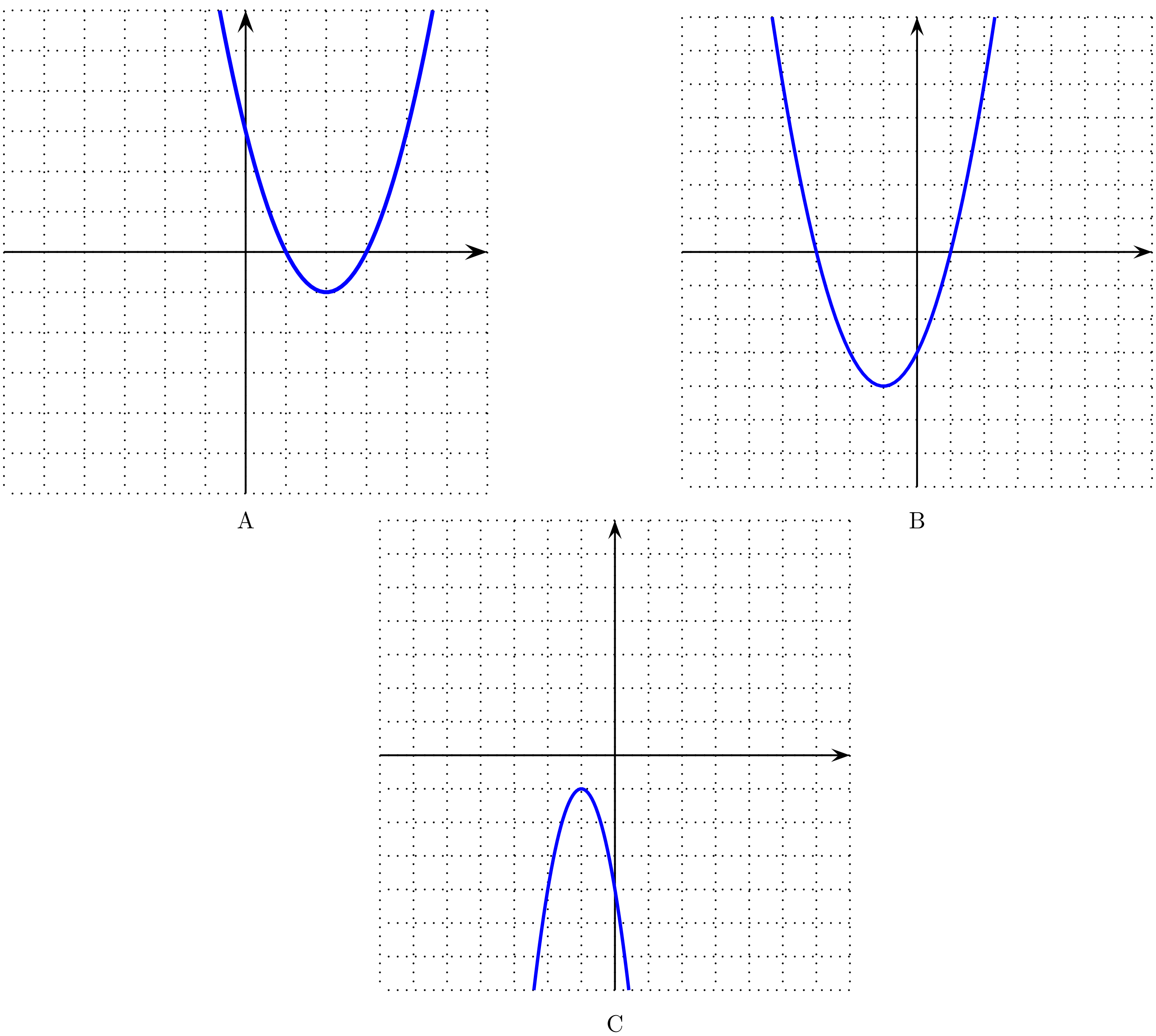


Figure 3: The graphs in Question 11

12. A. $y = -2(x+1)^2 + 5$ B. Domain is $(-\infty, \infty)$, Range is $(-\infty, 5]$ C. See Figure 4

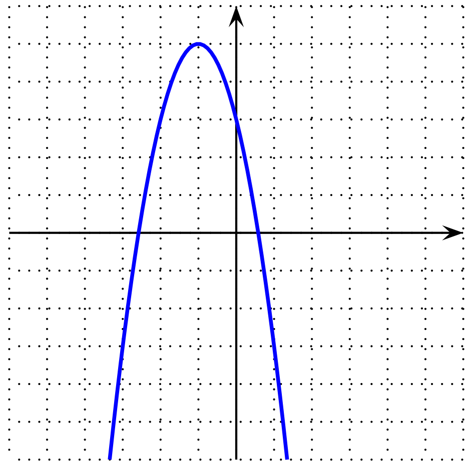


Figure 4: The parabola of Question 12

13. (a) A. $\{\pm 1, \pm 2, \pm 4\}$ B. $\{\pm 1, \pm 2, \pm 3, \pm 6\}$ C. $\{\pm 1, \pm 2, \pm \frac{1}{2}\}$
    (b) A. $(x+1)(x-2)^2$ B. $(x+1)(2-x)(x-3)$ C. $(x+1)^2(x+2)(2x-1)$
    (c) A. $x = -1$, $x = 2$ B. $x = -1$, $x = 2$, $x = 3$ C. $x = -1$, $x = -2$, $x = \frac{1}{2}$
    Answers to the remaining parts can be read from the graphs in Figure 5

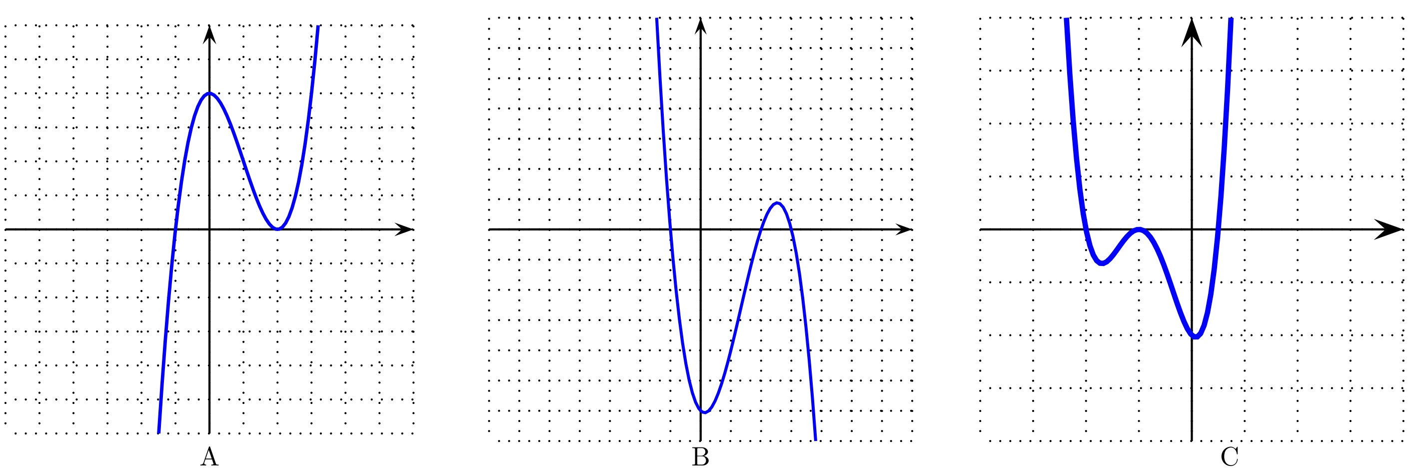


Figure 5: The graphs in Question 13

14. By the Remainder Theorem the answer is 42.

15. (a) A. $\dfrac{x+1}{x-2}$, $x \neq -1$ B. $\dfrac{(x+3)(x-1)}{(x+1)(x-3)}$ C. $\dfrac{(x+3)(x-3)}{(x-2)(x+1)}$ D. $\dfrac{2-x}{(x+2)(x-1)}$ E. $\dfrac{x^2}{x^2+1}$

Answers to the remaining parts can be read from the graphs in Figure 6 and Figure 7.

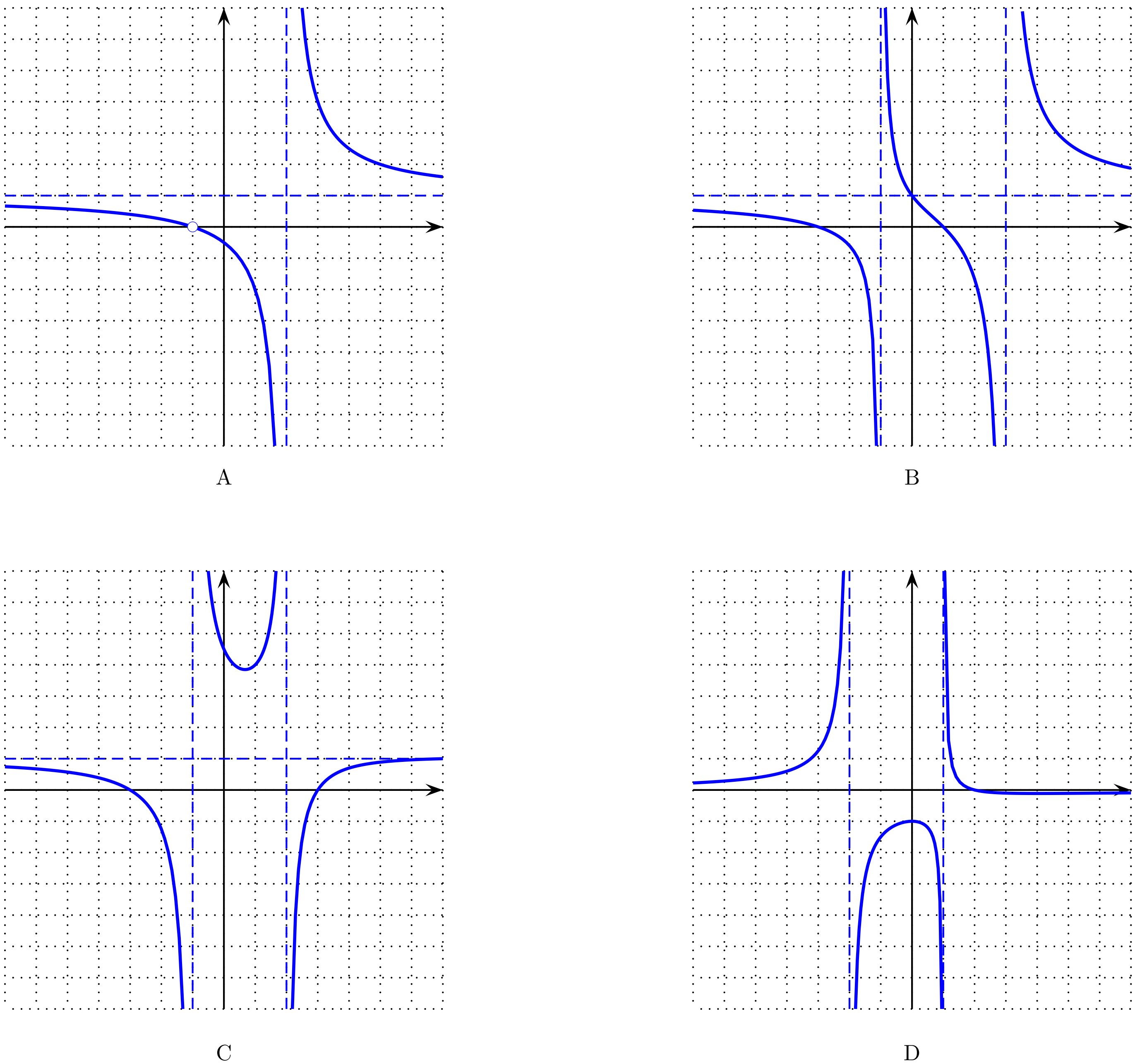


Figure 6: The first four graphs of Question 15

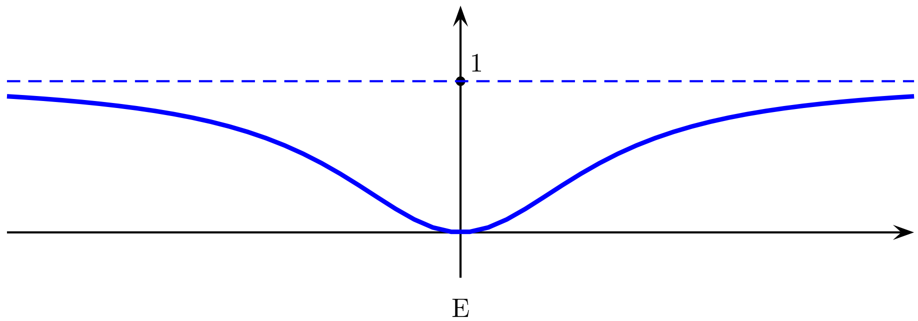


Figure 7: The fifth graph of Question 15

16. (a) A. $(-\infty, \infty)$ B. $(-\infty, \infty)$ C. $(0, \infty)$ D. $(-1, \infty)$
    (b) A. $(1, \infty)$ B. $(-4, \infty)$ C. $(-\infty, \infty)$ D. $(-\infty, \infty)$
    (c) A. $y = 1$ B. $y = -4$ C. $x = 0$ D. $x = -1$
    (d) See the graphs in Figure 8

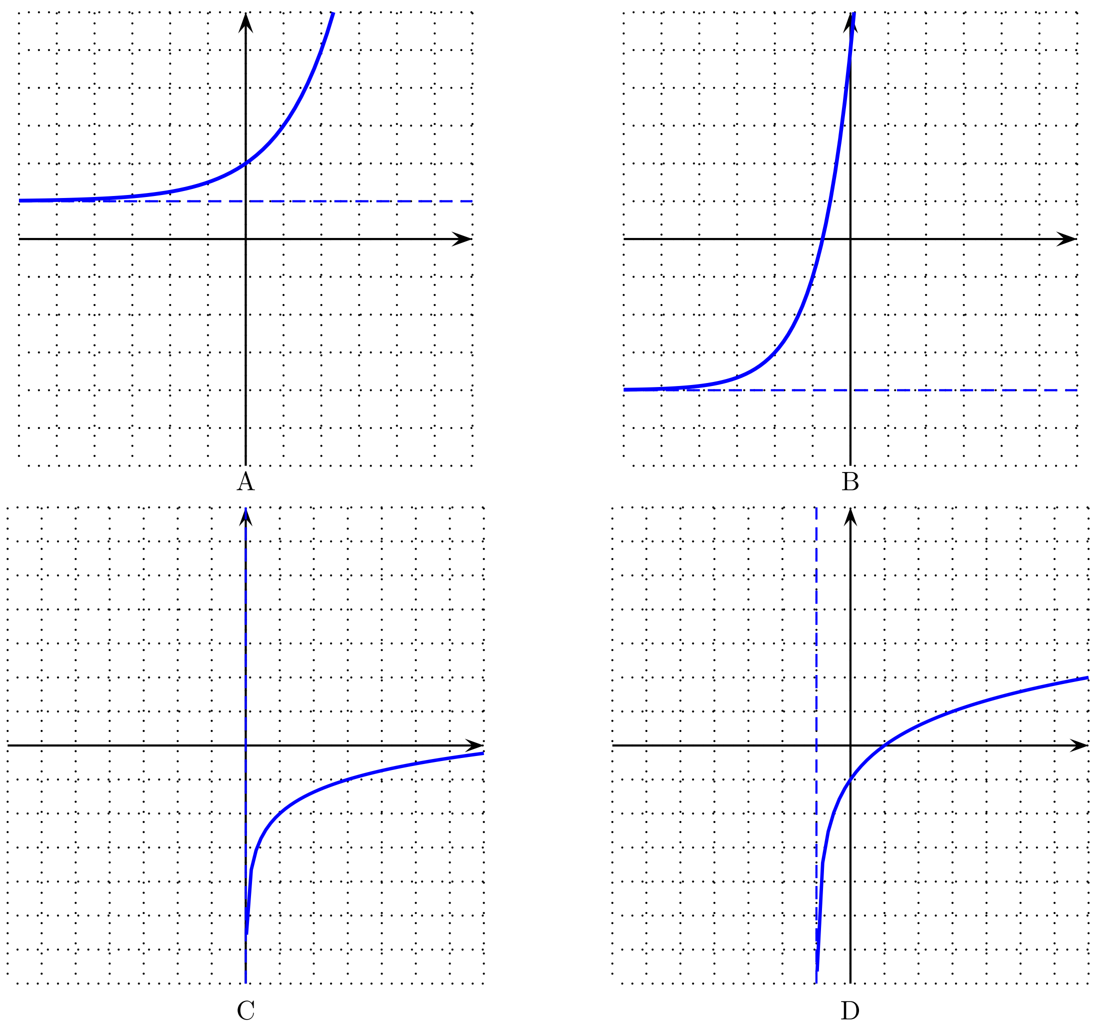


Figure 8: The graphs of Question 16

17. A. $(-\infty,-1)\cup(2,\infty)$ B. $\left[\frac{1}{3},1\right]$

18. A. -6 B. $-\frac{1}{4}$ C. 42 D. $\frac{3}{4}$ E. 8 F. $\sqrt{2}$ G. 3 H. 1.76 I. $-\frac{\sqrt{2}}{2}$ J. $-\frac{\sqrt{3}}{3}$ K. $\frac{\sqrt{3}}{2}$
L. -2 M. $\frac{\pi}{4}$ N. $\frac{2\pi}{3}$ O. $\frac{\pi}{6}$ P. $\frac{2\pi}{3}$ Q. 0 R. $\frac{3+8\sqrt{2}}{15}$

19. A. $\frac{11\pi}{6}$ B. $\frac{7\pi}{6}$ C. $\frac{3\pi}{4}$

20. A. $x=2$ B. $x=0$ C. $x=\frac{\pi}{3}$, $x=\frac{2\pi}{3}$, $x=\frac{4\pi}{3}$, $x=\frac{5\pi}{3}$ D. $x=\pi$, $x=\frac{2\pi}{3}$, $x=\frac{4\pi}{3}$

21. To prove these identities, use algebra and the basic identities

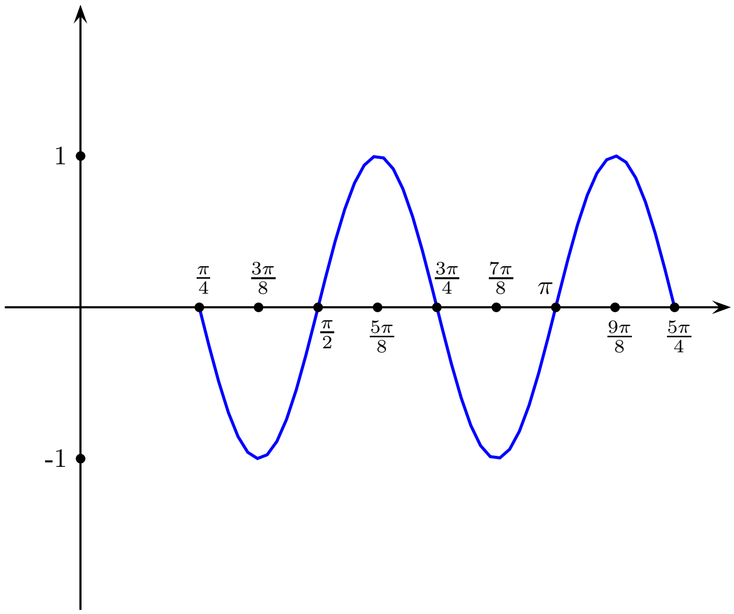
$$\begin{aligned}
\csc\theta &= \frac{1}{\sin\theta}\\
\sec\theta &= \frac{1}{\cos\theta}\\
\cot\theta &= \frac{1}{\tan\theta}\\
\tan\theta &= \frac{\sin\theta}{\cos\theta}\\
\cos^2\theta+\sin^2\theta &= 1
\end{aligned}$$

22. (a) A. $\frac{\pi}{2}$ B. π C. $\frac{2\pi}{3}$ D. 4π

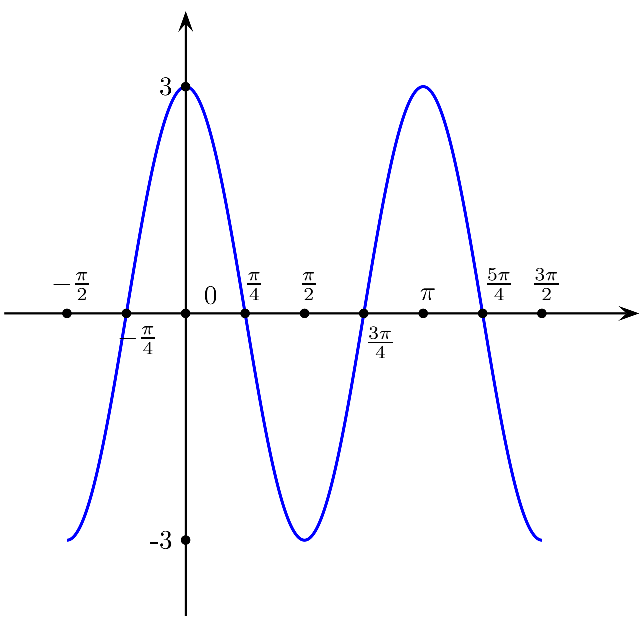
(b) A. 1 B. 3 C. 2 D. $\frac{1}{2}$

(c) A. $\frac{\pi}{4}$ B. $-\frac{\pi}{2}$ C. $\frac{\pi}{6}$ D. π

The graphs are as follows:

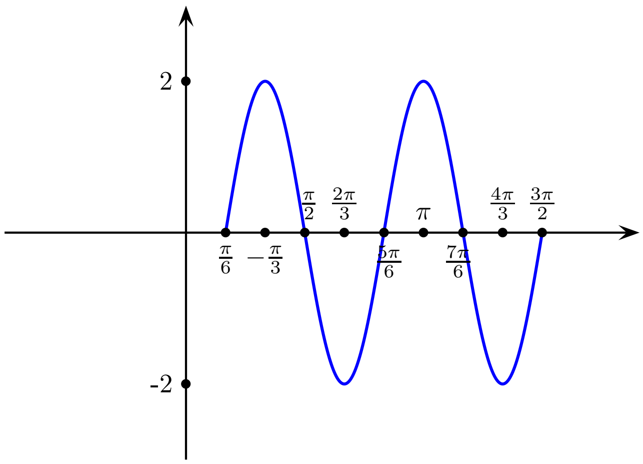


(a)

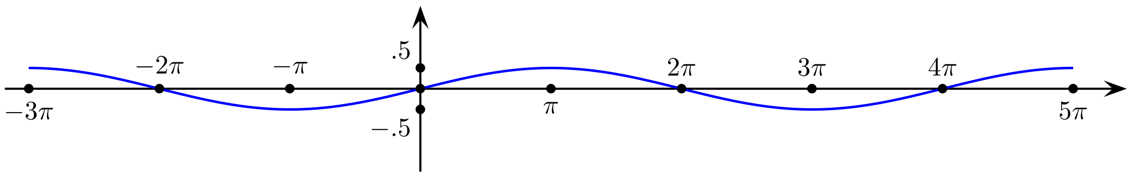


(b)

(c)



(d)



23. $-\frac{\sqrt{7}}{3}$

IP. YH, NEA, TK Apr 2022