# Graphical Examples of Cop-Win Graphs

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This document is a supplement to our paper *Corner Ranking, Realizable Vectors, and Extremal Cop-Win Graphs* [1]. In that paper a number of graphs that are minimal or conjectured to be minimal are described. Here we show figures of these graphs, which would have taken up too much space in the original paper. For the larger graphs, we also include a description of the graph by listing its edge set directly beneath the figure.

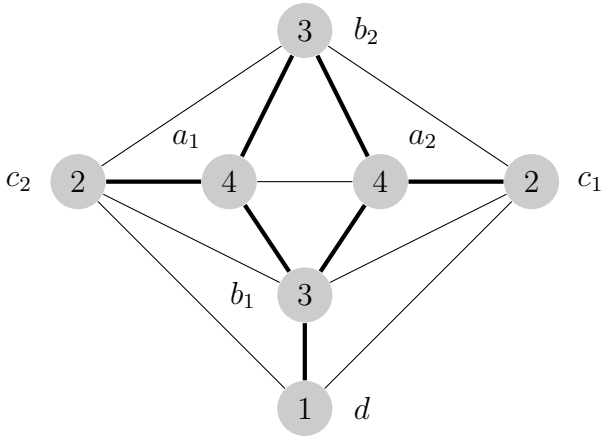
In each of the first four sections, in each subsection we show an example of a graph that realizes the given rank cardinality vector. Then in the Section 5 we show all the graphs realizing some small rank cardinality vectors.

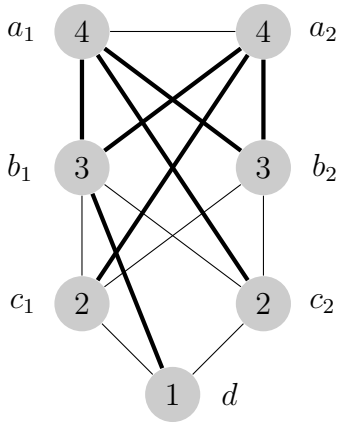
# Contents

# 1 Minimal 1-cop-win graphs

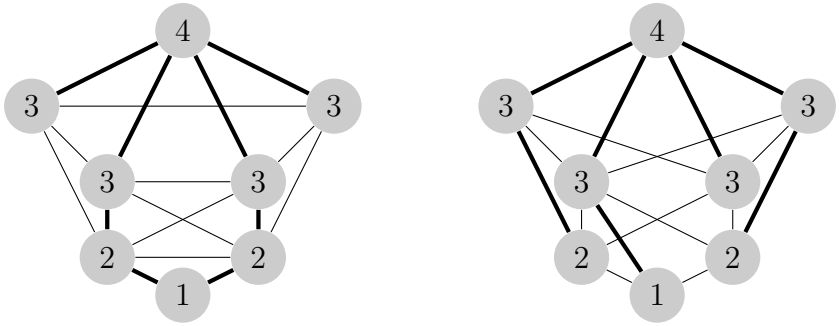
As described in [1], the three vectors that are known to be 1-minimal are $(1,2)$, $(2,2,2,1)$, and $(1,4,2,1)$. In this section we give graphical examples of 1-realizations of the last two of these.

## 1.1 Two representations of the unique 1-realization of (2,2,2,1)



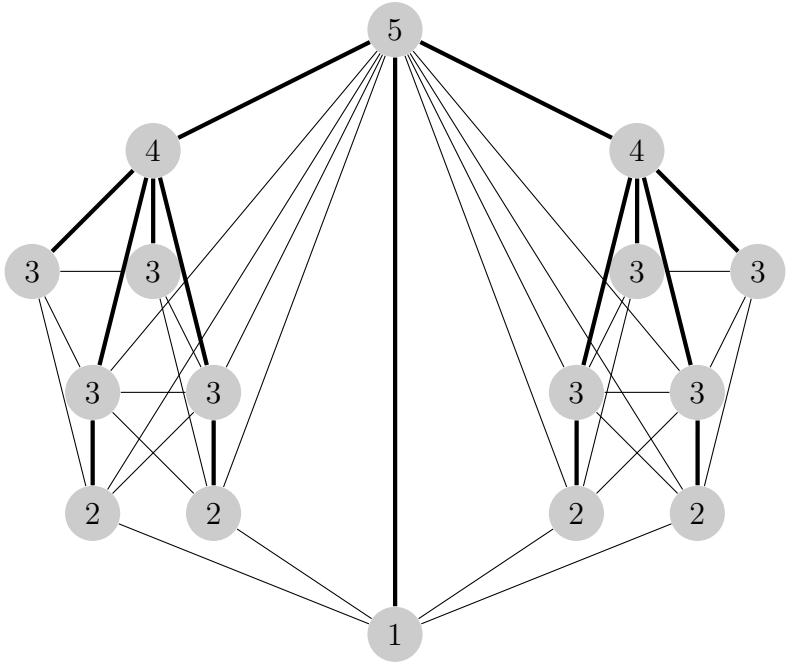
### 1.2 Two different graphs 1-realizing $(1, 4, 2, 1)$




## 2 1-cop-win graphs conjectured to be minimal

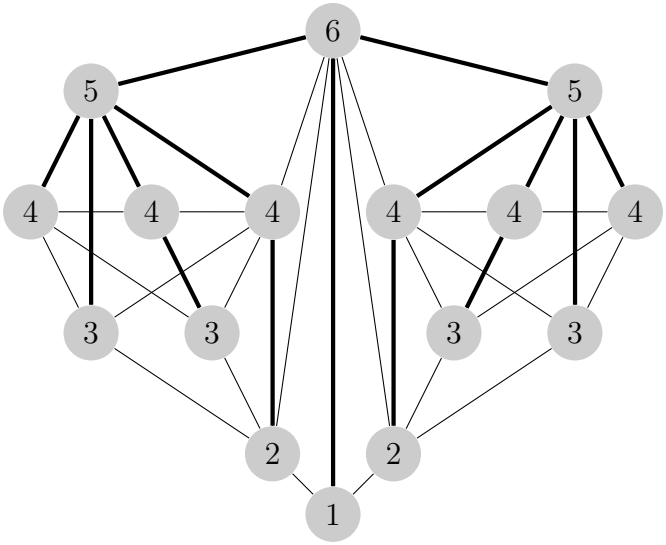
This section contains graphical examples of 1-cop-win graphs realizing vectors from [1] that are conjectured to be 1-minimal (see the end of that paper).

### 2.1 $(1, 2, 8, 4, 1)$



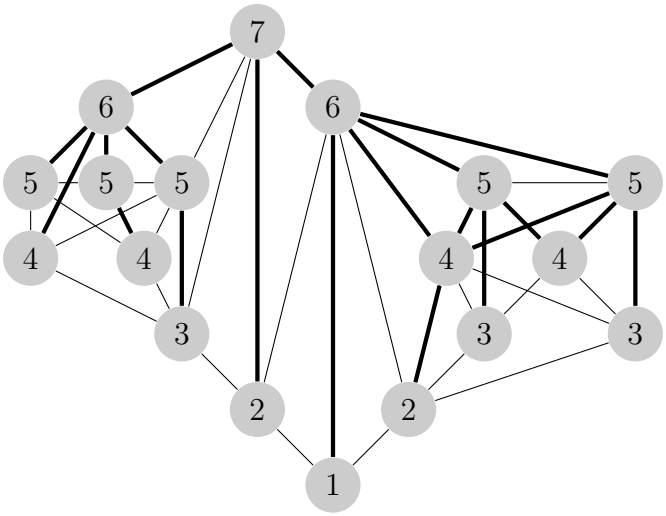

(15,4) (15,5) (15,8) (15,9) (3,4) (4,5) (5,6) (6,3) (7,8) (8,9)
(9,10) (10,7) (3,11) (4,11) (5,11) (15,11) (4,12) (5,12) (6,12) (15,12)
(7,13) (9,13) (8,14) (10,14) (15,13) (15,14) (11,16) (12,16) (13,16) (14,16)
(15,1) (15,2) (1,3) (1,4) (1,5) (1,6) (2,7) (2,8) (2,9) (2,10)
(8,13) (9,14) (15,16)

**2.2** $(1, 2, 6, 4, 2, 1)$



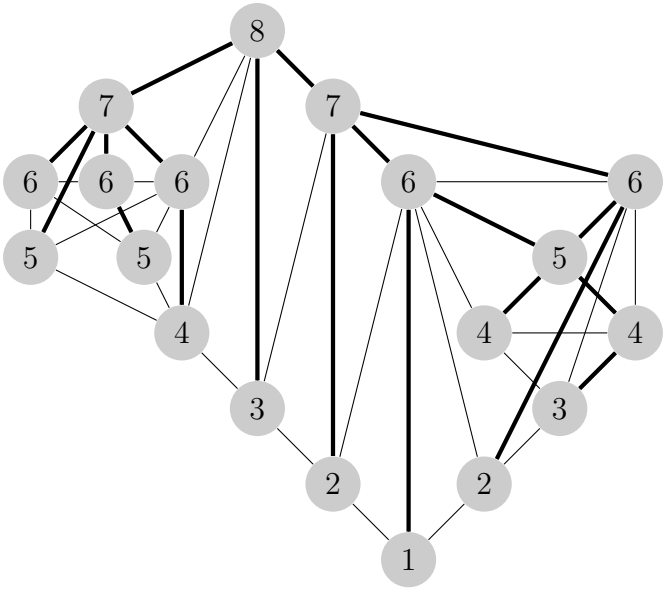

(1,2) (1,3) (2,4) (2,5) (2,6) (3,7) (3,8) (3,9) (4,5) (5,6)
(7,8) (8,9) (2,11) (4,11) (6,11) (4,12) (5,12) (6,12) (7,14) (9,14)
(3,14) (8,13) (9,13) (7,13) (13,10) (14,10) (12,16) (11,16) (16,15) (10,15)
(1,15) (1,16) (1,10) (1,6) (1,7) (6,16) (7,10)

**2.3** $(1, 2, 5, 4, 3, 2, 1)$



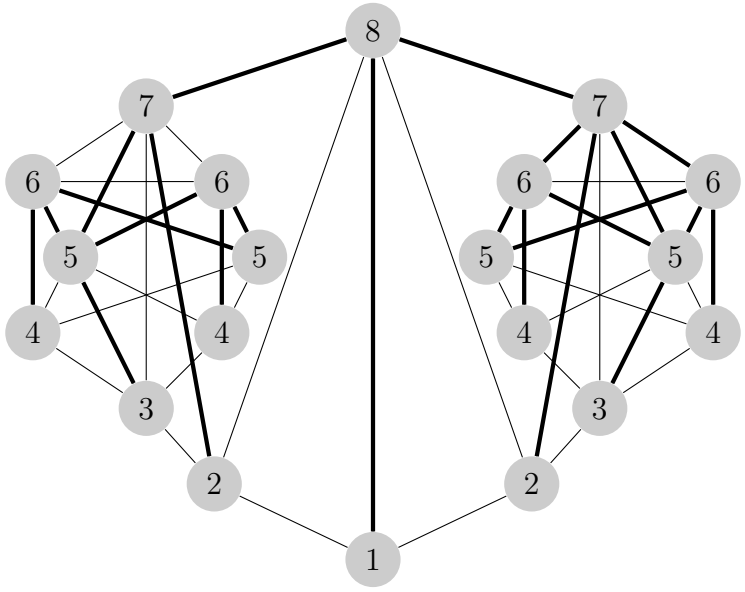

(1,2) (1,3) (1,6) (1,16) (2,4) (2,5) (2,6) (2,9) (3,7) (3,8)
(3,12) (3,17) (3,18) (3,16) (4,5) (5,6) (7,8) (4,9) (4,10) (5,10)
(6,9) (6,10) (6,13) (9,13) (10,13) (13,16) (16,18) (7,11) (7,12) (7,14)
(8,11) (8,12) (8,15) (11,14) (11,15) (12,14) (12,15) (12,17) (14,17) (15,17)
(17,18) (1,13)

**2.4** $(1, 2, 5, 3, 3, 2, 2, 1)$



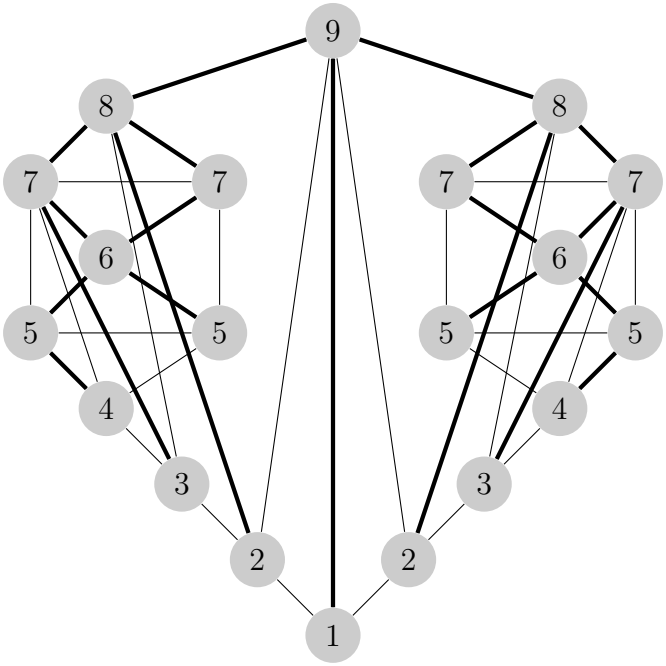

(1,2) (1,3) (1,12) (1,15) (1,6) (2,4) (2,5) (2,6) (3,7) (3,8)
(3,15) (3,17) (2,9) (4,5) (5,6) (7,8) (4,9) (4,10) (6,12) (5,10)
(6,9) (6,10) (7,11) (7,13) (7,17) (7,19) (7,18) (8,11) (8,14) (8,16)
(8,18) (9,12) (10,12) (11,13) (11,14) (12,15) (13,14) (13,16) (14,16) (15,17)
(16,18) (17,19) (18,19)

**2.5** $(1, 2, 4, 4, 4, 2, 2, 1)$



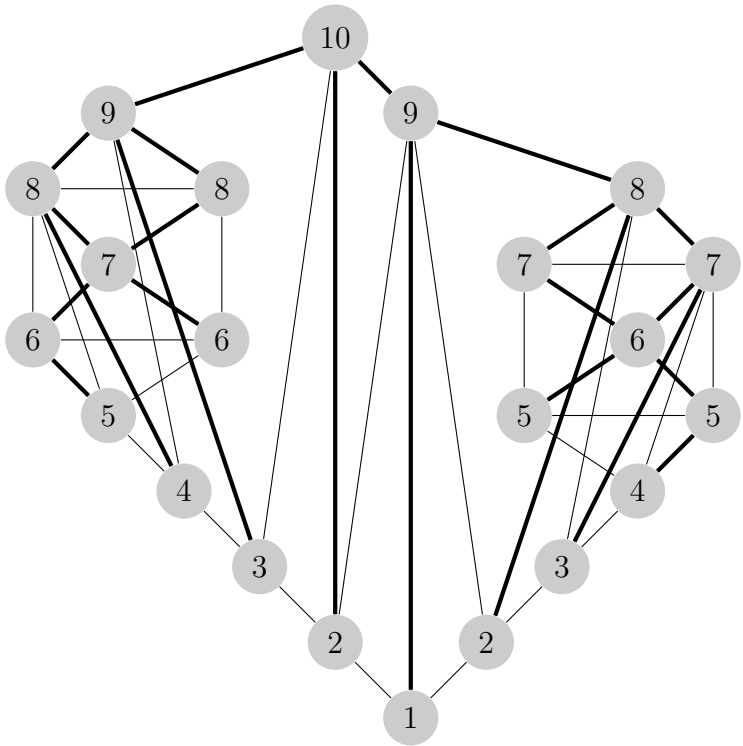

(1,2) (1,3) (1,18) (1,19) (1,20) (2,4) (2,5) (2,8) (2,16) (2,18)
(3,6) (3,7) (3,11) (3,17) (3,19) (4,5) (4,8) (4,12) (4,9) (5,8)
(5,9) (5,13) (6,7) (6,10) (6,14) (6,11) (7,10) (7,11) (7,15) (8,12)
(8,16) (8,13) (9,12) (9,13) (10,14) (10,15) (11,14) (11,15) (11,17) (12,16)
(13,16) (14,17) (15,17) (16,18) (17,19) (18,20) (19,20)

**2.6** $(1, 2, 4, 2, 4, 2, 2, 2, 1)$



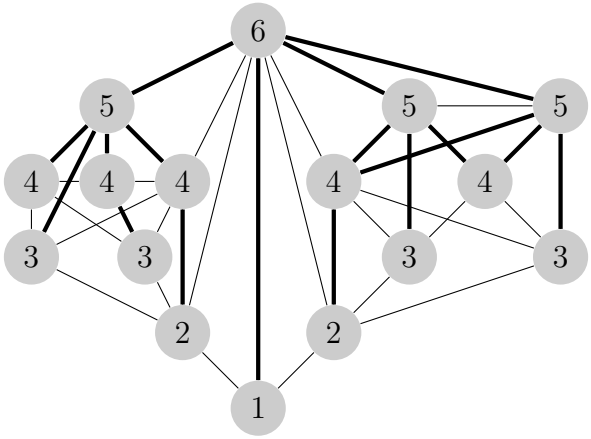

(1,2) (1,3) (1,18) (1,19) (1,20) (2,4) (2,5) (2,16) (2,18) (3,6)
(3,7) (3,17) (3,19) (4,5) (4,10) (4,8) (5,8) (5,11) (5,14) (5,16)
(6,7) (6,12) (6,9) (7,9) (7,13) (7,15) (7,17) (8,10) (8,11) (9,12)
(9,13) (10,11) (10,14) (11,14) (12,13) (12,15) (13,15) (14,16) (15,17) (16,18)
(17,19) (18,20) (19,20)

**2.7** $(1, 2, 3, 3, 3, 3, 2, 2, 2, 1)$



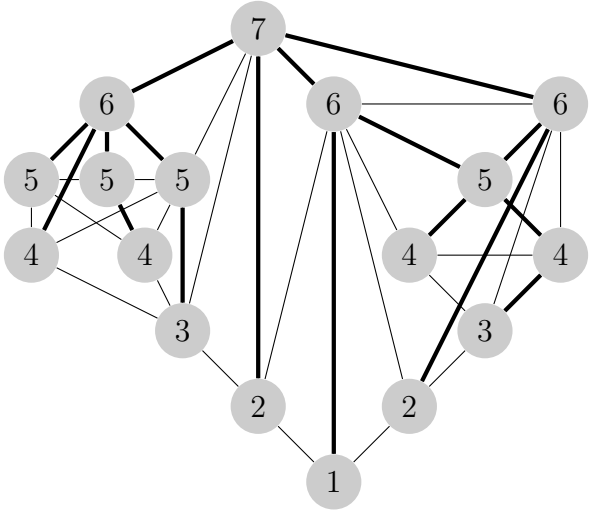
(20,22) (21,22) (17,19) (18,20) (19,21) (15,17) (13,16) (16,18) (12,14) (12,15)
(14,15) (14,17) (10,11) (10,13) (11,13) (9,12) (9,15) (9,17) (9,19) (7,10)
(7,11) (8,9) (8,12) (8,14) (6,8) (6,9) (6,19) (6,21) (5,7) (5,11)
(4,13) (4,16) (4,5) (4,10) (4,7) (1,2) (1,3) (1,20) (1,18) (2,4)
(2,5) (2,16) (2,18) (3,6) (3,20) (3,21) (3,22)

**2.8** $(1, 3, 5, 4, 2, 1)$



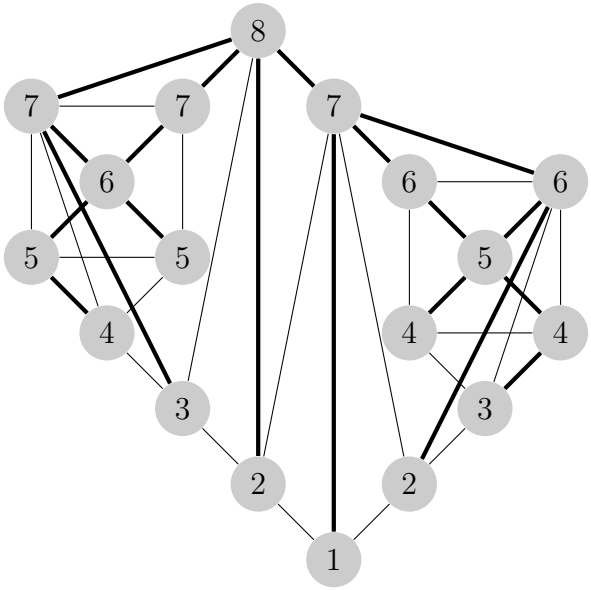

(1,2) (1,3) (1,4) (1,7) (1,9) (1,14) (1,15) (1,16) (2,5) (2,6)
(2,7) (2,10) (5,6) (6,7) (5,10) (5,11) (6,11) (7,10) (7,11) (7,14)
(10,14) (11,14) (14,16) (3,4) (3,8) (3,9) (3,12) (4,8) (4,9) (4,13)
(8,12) (8,13) (9,12) (9,13) (9,15) (12,15) (13,15) (15,16)

**2.9** $(1, 3, 4, 4, 2, 2, 1)$




(1,2) (1,3) (1,4) (1,13) (1,15) (1,7) (2,5) (2,6) (2,7) (2,9)
(5,6) (6,7) (5,9) (5,10) (6,10) (7,9) (7,10) (7,13) (9,13) (10,13)
(13,15) (15,17) (3,4) (3,8) (3,11) (3,16) (3,17) (3,15) (4,8) (4,12)
(4,14) (4,16) (8,11) (8,12) (11,12) (11,14) (12,14) (14,16) (16,17)

**2.10** $(1, 3, 3, 3, 3, 2, 2, 1)$

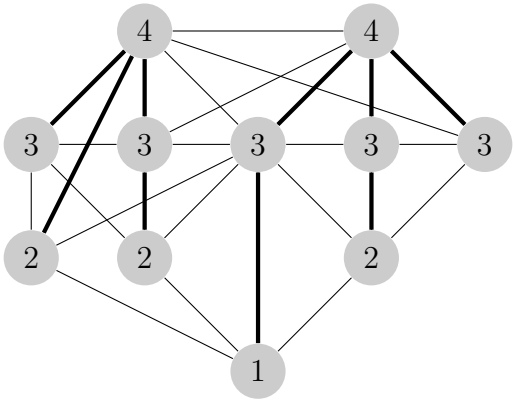



(1,2) (1,3) (1,4) (1,14) (1,16) (2,3) (2,5) (2,8) (3,5) (3,9)
(3,11) (3,14) (4,16) (4,6) (4,7) (4,17) (4,18) (5,8) (5,9) (6,7)
(6,12) (6,10) (7,10) (7,13) (7,15) (7,17) (8,9) (8,11) (9,11) (10,12)
(10,13) (11,14) (12,13) (12,15) (13,15) (14,16) (15,17) (16,18) (17,18)

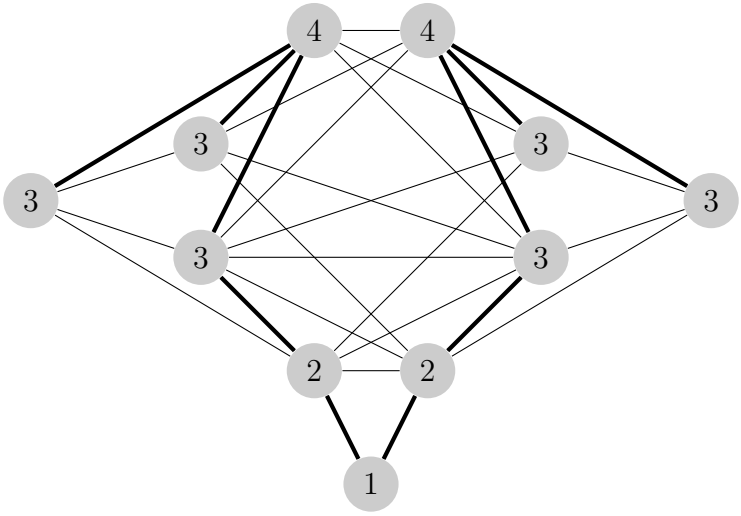
# 3 Minimal 0-cop-win graphs

As described in [1], the four vectors that are known to be 0-minimal are $(2, 2)$, $(2, 5, 3, 1)$, $(2, 6, 2, 1)$, and $(3, 3, 2, 1)$. In this section we give examples of 0-cop-win graphs realizing the last three of these.

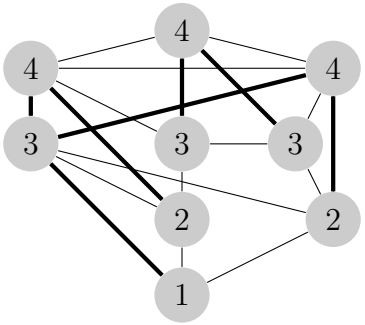
**3.1** $(2, 5, 3, 1)$



**3.2** $(2, 6, 2, 1)$



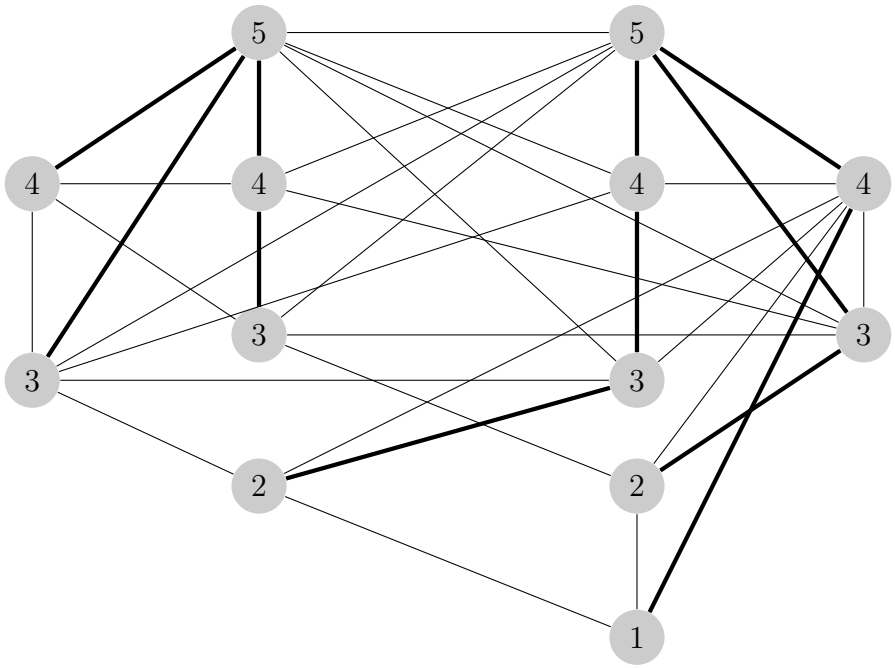

**3.3** $(3, 3, 2, 1)$




# 4 0-cop-win graphs conjectured to be minimal

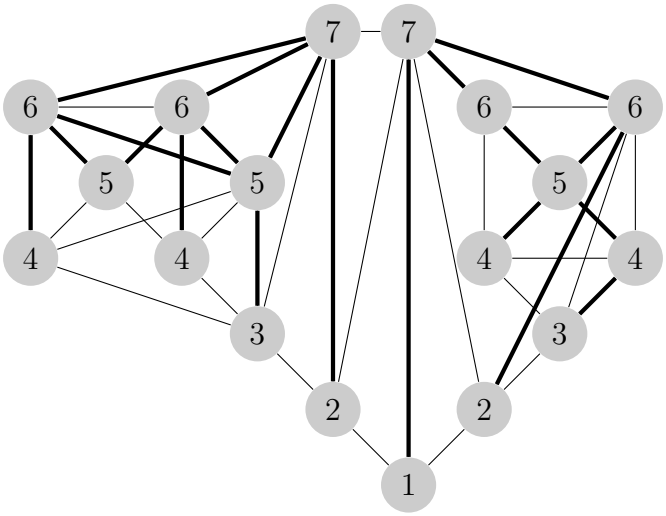
This section contains graphical examples of 0-cop-win graphs realizing vectors from [1] that are conjectured to be 0-minimal (see the end of that paper).

**4.1** $(2, 4, 4, 2, 1)$



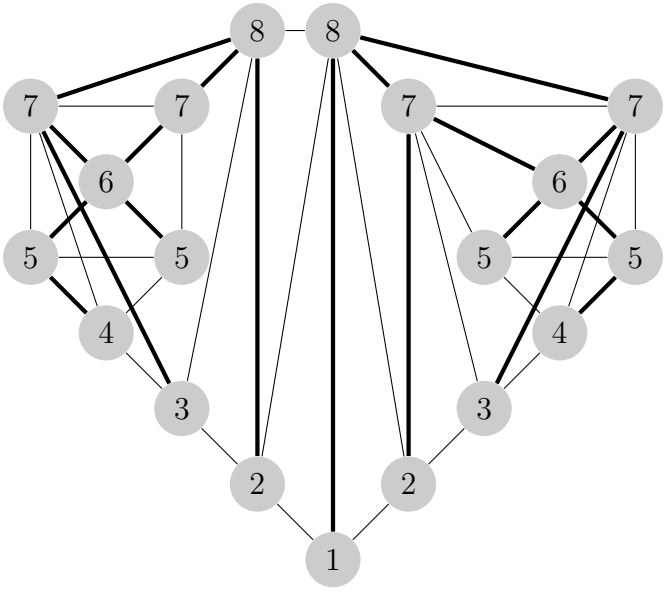

(4,8) (4,9) (4,11) (4,5) (8,11) (8,5) (8,12) (9,11) (5,12) (4,13)
(4,2) (8,2) (8,1) (5,2) (9,2) (9,1) (11,1) (5,13) (12,13) (12,10)
(8,10) (4,10) (11,10) (2,13) (1,10) (2,3) (13,3) (12,3) (1,6) (10,6)
(12,6) (3,7) (6,7) (12,7)

**4.2** $(2, 4, 3, 4, 2, 2, 1)$



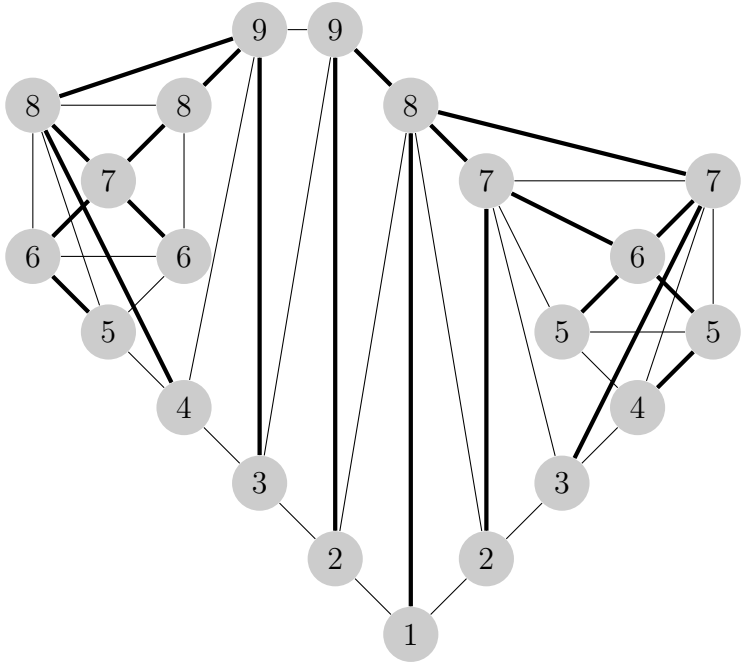

(1,2) (1,3) (1,4) (1,7) (1,14) (1,16) (2,5) (2,6) (2,16) (2,17)
(2,18) (3,4) (3,7) (3,10) (3,8) (4,7) (4,8) (4,11) (5,6) (5,12)
(5,9) (6,9) (6,13) (6,15) (6,17) (7,10) (7,11) (7,14) (8,10) (8,11)
(9,12) (9,13) (10,14) (11,14) (12,13) (12,15) (13,15) (14,16) (15,17) (16,18)
(17,18)

**4.3** $(2, 4, 2, 4, 2, 2, 2, 1)$



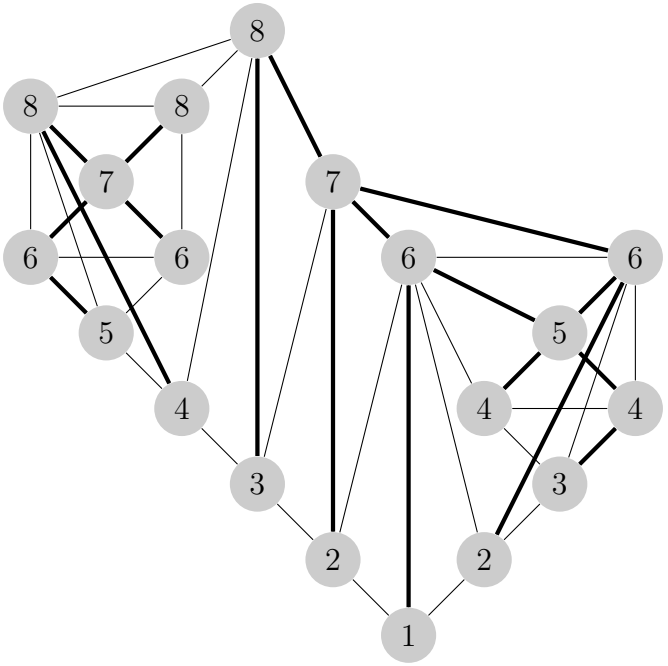

(1,2) (1,3) (1,4) (1,15) (1,17) (2,5) (2,6) (2,17) (2,18) (2,19)
(3,4) (3,7) (3,9) (3,13) (3,15) (4,7) (4,10) (5,6) (5,8) (5,11)
(5,16) (5,18) (6,8) (6,12) (6,14) (6,16) (7,9) (7,10) (8,11) (8,12)
(9,13) (10,13) (11,14) (12,14) (9,10) (11,12) (13,15) (14,16) (15,17) (16,18)
(17,19) (18,19)

**4.4** $(2, 3, 3, 3, 3, 2, 2, 2, 1)$



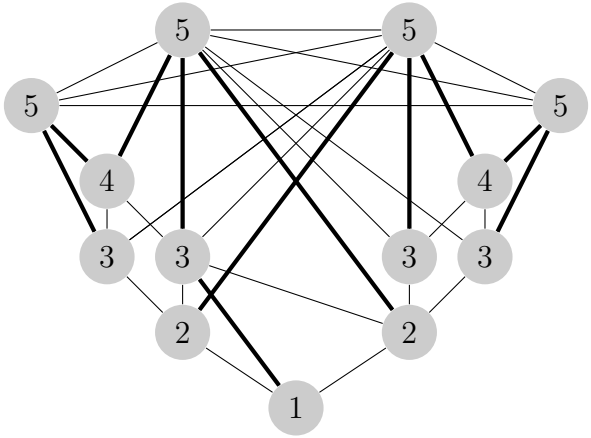

(1,2) (1,3) (1,4) (1,15) (1,17) (2,5) (2,17) (2,19) (3,4) (3,6)
(3,9) (4,6) (4,12) (4,15) (4,10) (5,7) (5,8) (5,19) (5,20) (5,21)
(6,9) (6,10) (7,8) (7,13) (7,11) (8,11) (8,14) (7,18) (7,20) (8,16)
(8,18) (9,12) (10,12) (9,10) (11,13) (11,14) (12,15) (13,14) (13,16) (14,16)
(15,17) (16,18) (17,19) (18,20) (19,21) (20,21)

**4.5** $(3, 2, 4, 2, 3, 2, 2, 1)$




(1,2) (1,3) (1,4) (1,7) (2,4) (2,3) (2,10) (2,12) (2,6) (4,6)
(4,7) (6,7) (6,10) (7,10) (10,12) (12,15) (15,17) (17,19) (3,5) (3,12)
(3,15) (5,8) (5,9) (5,15) (5,17) (8,9) (8,13) (8,17) (8,18) (8,19)
(8,11) (9,11) (9,14) (9,16) (9,18) (11,13) (11,14) (13,14) (13,16) (14,16)
(16,18) (18,19)

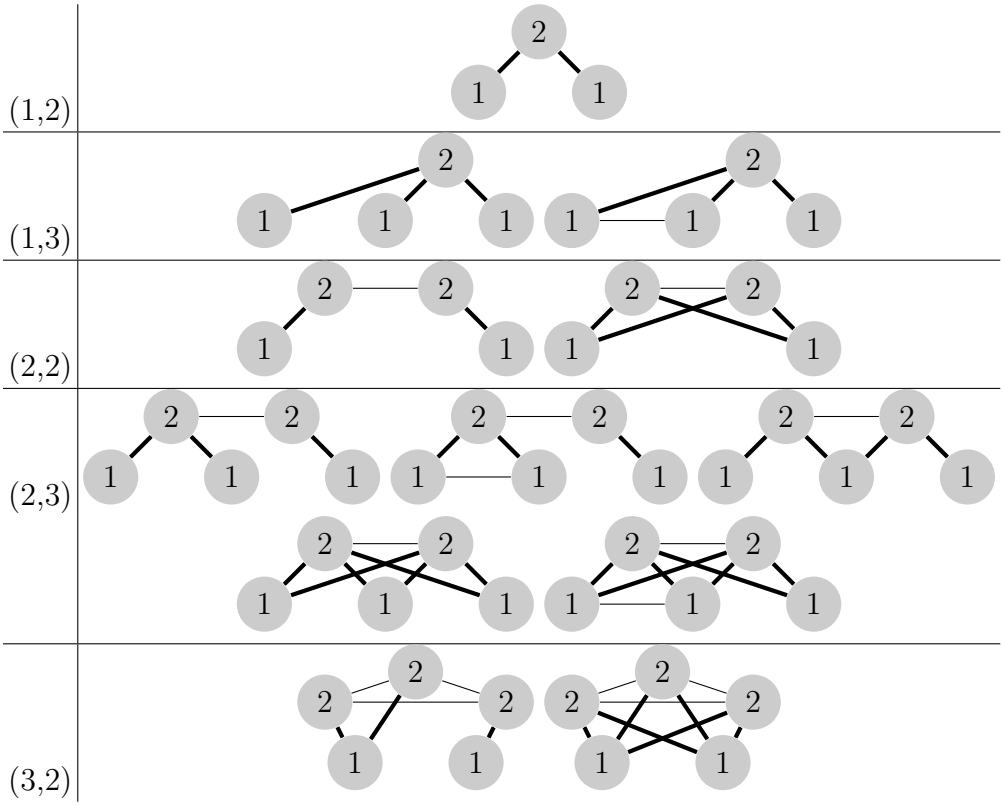
**4.6** $(4, 2, 4, 2, 1)$




(1,2) (1,3) (1,4) (2,3) (2,4) (3,4) (1,5) (1,7) (3,7) (2,5)
(2,8) (2,9) (2,10) (2,12) (5,7) (5,8) (3,6) (3,9) (3,7) (3,8)
(3,11) (4,6) (4,10) (6,9) (6,10) (7,11) (8,11) (8,12) (9,12) (10,12)
(11,13) (12,13) (8,13)

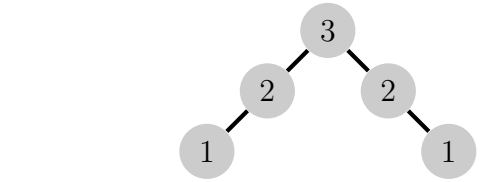
# 5 Some Rank 2 and 3 Graphs

## 5.1 All graphs that realize some small rank 2 vectors

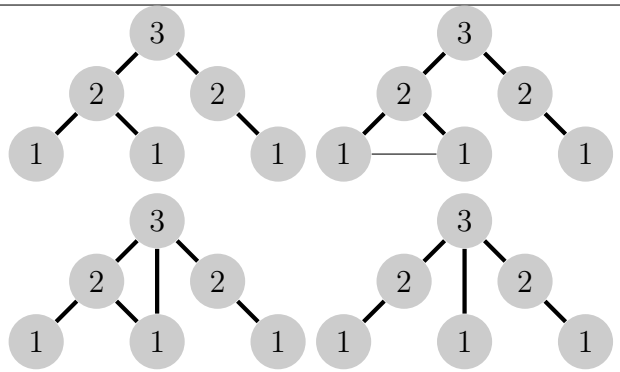


## 5.2 All graphs that realize some small rank 3 vectors

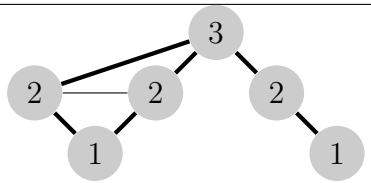
(1,2,2)



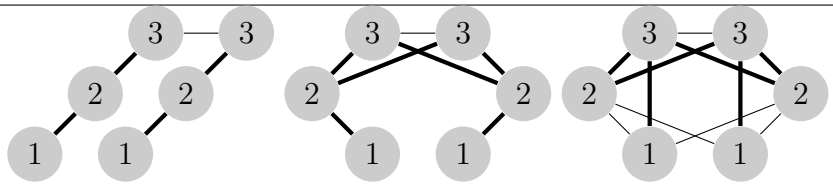

(1,2,3)



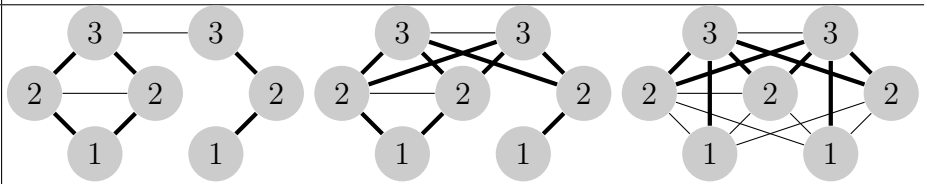

(1,3,2)



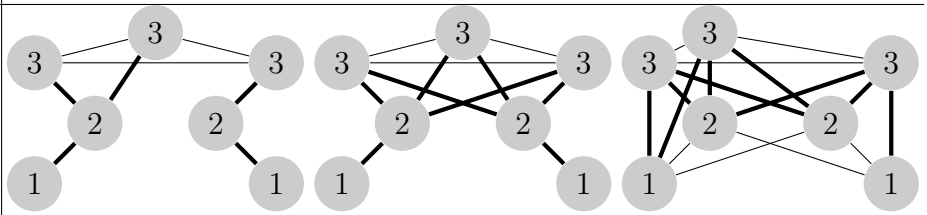

(2,2,2)



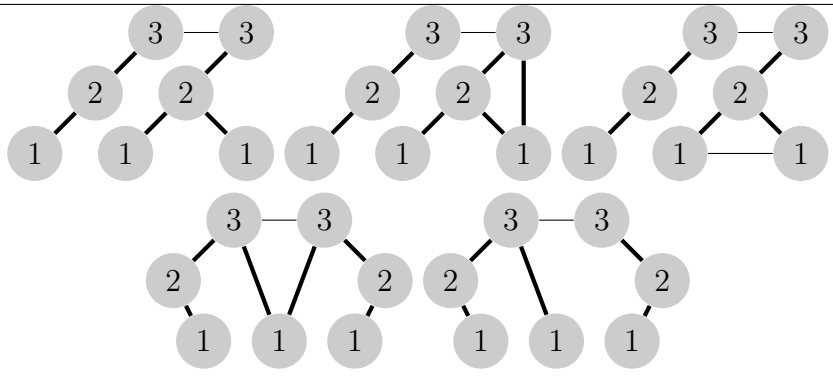

(2,3,2)




(3,2,2)




Graphs 0-realizing (2,2,3)



# References

[1] D. Offner and K. Ojakian, Corner Ranking, Realizable Vectors, and Extremal Cop-Win Graphs, submitted, 2017.