Math 46 Abstract Algebra

Homework 3: Symmetric and alternate groups. Due date: 04/17/2023

1. Write the following permutations in cycle notation:

$$(a)\begin{pmatrix}1&2&3&4&5\\2&4&1&5&3\end{pmatrix} \qquad (b)\begin{pmatrix}1&2&3&4&5\\1&4&3&2&5\end{pmatrix} \qquad (c)\begin{pmatrix}1&2&3&4&5\\3&5&1&4&2\end{pmatrix}$$

2. Simplify the following permutations and write them as products of disjoint cycles:

$(a)\,(12)(1234)$ $(b)\,(13)(24)(12)$ $(c)\,(132)(13)(123)$ $(d)\,(142)(35)(23)(152)$.

3. Simplify the following permutations and write them as products of disjoint cycles:

$(a)\,(12)(123)^2$ $(b)\,(142)^{-1}$ $(c)\,(16352)^3$ $(d)\,(1,10,9,7,6)(2,8,4)(3,5)^{10}$ $(e)\,(571)^8$.

4. Consider the 12-cycle $\sigma = (1\,2\,3\,4\,5\,6\,7\,8\,9\,10\,11\,12)$. For what positive integers i is σ^i again a 12-cycle?

5. Give an example of an element of A_{10} of order 15.

6. What is the maximum value for an order of an element in S_7?

7. Find all orders of elements in the group S_3. Find the conjugacy classes. Hint: Find all cycles types in S_3.

8. Find all orders of elements in the alternate group A_4. Find the conjugacy classes. Hint: Find all cycles types in S_4 and then check if elements remain conjugate in A_4.