DARTVII Titles and Abstracts.

Michael Semenov-Tian-Shansky
Université de Bourgogne
Title: *Poisson sturcture for differential/difference Lax equations and differential Galois theory.*
Abstract:
We discuss the transfer of the Poisson structure in the space of differential/difference Lax operators to the space of solutions of the corresponding auxiliary linear problem (the space of wave functions). A peculiar symmetry breaking gives rise to a nontrivial Poisson structure on the associated Galois group.

Michael Singer
North Carolina State University
Title: *Finite automata, automatic sets, and difference equations*
Abstract:
A finite automaton is one of the simplest models of computation. Initially introduced by McCulloch and Pitts to model neural networks, they have been used to aid in software design as well as to characterize certain formal languages and number-theoretic properties of integers. A set of integers is said to be m-automatic if there is a finite automaton that decides if an integer is in this set given its base-m representation. For example powers of 2 are 2-automatic but not 3-automatic. This latter result follows from a theorem of Cobham describing which sets of integers are m- and n-automatic for sufficiently distinct m and n. In recent work with Reinhard Schaefke, we gave a new proof of this result based on analytic results concerning normal forms of systems of difference equations. In this talk, I will describe this circle of ideas.

Anand Pillay
Notre Dame University
Title: *Picard Vessiot closed differential fields and constrained cohomology.*
Abstract:
We prove that a differential field K is both algebraically closed and Picard-Vessiot closed iff $H^1_c(K, G) = 1$ for any linear differential algebraic group over K.

Julia Hartmann
University of Pennsylvania
Title: *Differential Embedding Problems over Complex Function Fields.*
Abstract:
Differential Galois theory studies differential field extensions generated by solutions of differential equations by means of their symmetry groups. The question which groups can occur in this context is known as the inverse problem. Differential embedding problems can be considered as a refinement if this question. They are used to study how Picard-Vessiot extensions - and their differential Galois groups - fit together in towers. In this talk, we show that over a complex function field in one variable, every embedding problem has a proper solution. The key ingredient is a patching result for differential torsors. This is joint work with A. Bachmayr, D. Harbater, and M. Wibmer.

Richard Gustavson
Graduate Center CUNY
Title: *Differential Kernels and Bounds for the Consistency of Differential Equations*
Abstract:
A differential kernel is a field extension of a differential field that encodes certain possible solutions to a system of polynomial differential equations. Determining if that possible solution is an actual solution is then equivalent to asking whether a regular realization of the differential kernel exists. Differential kernels were first studied by Lando in 1970 in the case of a single derivation, in which case every differential kernel has a regular realization. With more than one derivation, however, regular realizations do not always exist. In 2014, Pierce computed an upper bound for the number of prolongations of a given differential kernel needed to ensure the existence of a regular realization. In this talk, I will present an improved upper bound for the number of prolongations needed. This effective bound leads to significantly better results in the case

of small numbers of derivations, and has applications to other areas of effective differential algebra, such as improved upper bounds for the order of characteristic sets of prime differential ideals and the effective differential Nullstellensatz. This is joint work with Omar Leon Sanchez.

Andrei Gabrielov
Purdue University
Title: *Classification of spherical and circular quadrilaterals*
Abstract:
A spherical polygon (membrane) is a bordered surface homeomorphic to a closed disc, with n distinguished boundary points called corners, equipped with a Riemannian metric of constant curvature 1, except at the corners where it has conical singularities, and such that the boundary arcs between the corners are geodesic. Spherical polygons with $n = 3$ and $n = 4$ are called spherical triangles and quadrilaterals, respectively.

This is a very old problem, related to the properties of real solutions of the real Heun equation (a Fuchsian differential equation with four real singular points and real coefficients). The corresponding problem for generic spherical triangles, related to the hypergeometric equation, was solved by Felix Klein more than 100 years ago, while non-generic cases were completely classified as late as 2011.

When all interior angles at the corners are integer multiples of π, classification of spherical quadrilaterals is equivalent to classification of rational functions with four real critical points, a special case of the B. and M. Shapiro conjecture.

Rational functions with real critical points can be characterized by their nets, combinatorial objects similar to Grothendieck's dessins d'enfants. Similarly, spherical polygons can be characterized by multi-colored nets, non-algebraic analogs of dessins d'enfants.

If time permits, I'll tell about classification of circular quadrilaterals (with the sides mapped to not necessarily geodesic circles on the sphere), and about connection between isomonodromic deformation of a Fuchsian differential equation with 5 singular points and solutions of the Painleve VI equation. This connection allows one to represent a real solution of the real Painleve VI equation by a sequence of nets of circular quadrilaterals and special (with one angle equal to 2π) circular pentagons.

Amador Martin-Pizarro
C.N.R.S. Institut Camille Jordan Lyon
Title: *On bounded automorphisms of fields with operators.*
Abstract:
Lascar showed that the group of automorphisms of the complex field which fix the algebraic closure of the prime field is simple. For this, he first showed that there is no non-trivial bounded automorphisms. An automorphism τ is bounded if there is a finite set A such that the image $\tau(b)$ of every element b is algebraic over $A \cup \{b\}$. The same result holds for a saturated differentially closed field of characteristic zero, where we replace algebraic by differentially algebraic. Together with T. Blossier and C. Hardouin, we provide a complete classification of bounded automorphisms in various fields equipped with operators, among others, for generic difference fields in all characteristics or for Hasse-Schmidt differential fields in positive characteristic.

Thomas Dreyfus
Université Lyon 1
Title: *Computing the differential Galois group using reduced forms.*
Abstract:
To a linear differential system one may associate a group, the differential Galois group, that measures the algebraic relations between the solutions of the system. The latter may be seen as an algebraic group and its computation is in general a difficult task. In this talk we explain how to transform the system so that the new system belong to the Lie algebra of the differential Galois group (such transformation always exists and the new system will be said to be on the Kolchin-Kovacic reduced form). Furthermore, as we will see, the latter reduction will be helpful for computing the differential Galois group. This is a joint work with Jacques-Arthur Weil.

Uma Iyer
Bronx Community College CUNY
Title: *Representations of an algebra of quantum differential operators.*
Abstract:
The algebra of quantum differential operators on graded algebras was introduced by V. Lunts and A. Rosenberg.

D. Jordan, T. McCune and Iyer have identified this algebra of quantum differential operators on the polynomial algebra with coefficients in an algebraically closed field of characteristic zero. It contains the first Weyl algebra and the quantum Weyl algebra as its subalgebras. In this talk we present the classification of irreducible weight modules over the algebra of quantum differential operators over the polynomial algebra in one variable. This is joint work with V. Futorny.

Gal Binyamini
Weizmann Institute of Science
Title: *Density of rational points on transcendental varieties.*
Abstract:
I will discuss results connected to the asymptotic density of rational points on transcendental varieties, starting with the theorem of Pila-Wilkie on general o-minimal structures and some of its diophantine applications. I will then discuss the Wilkie conjecture asserting a significant sharpening of the P-W theorem for the structure R_exp, and a recent proof (joint with Novikov) of this conjecture for sets definable using only restricted exponentiation (and restricted sine). I will briefly discuss the proof, and in particular the significant role played by the algebraic differential equations for exp and sin. Finally, I will discuss some related questions of diophantine significance for structures generated by elliptic, modular and other special functions, and what role the differential-algebraic structure of these functions may play in resolving these questions.

Carlos Arreche
North Carolina State University
Title: *Projectively integrable linear difference equations and their Galois groups.*
Abstract:
A difference-differential field (of characteristic 0) is a field k equipped with an automorphism σ and a derivation δ that commute with each other. A linear difference equation is integrable if its solutions also satisfy a linear differential system of the same size. The difference equation is projectively integrable if it becomes integrable "after moding out by scalars". Based on recent results of R. Schaefke and M. Singer, we show that when $k = C(x)$ and σ is either a shift, q, or Mahler operator, the difference-differential Galois group G attached to a projectively integrable difference equation has a very special form. These results have applications for the direct problem of computing G for a given linear difference equation, as well as for the inverse problem of deciding which linear differential algebraic groups occur as difference-differential Galois groups for such equations. This is joint work with Michael Singer.

James Freitag
University of Illinois at Chicago
Title: *Ranks in differential fields.*
Abstract:
We will introduce various model theoretic ranks on differential varieties, and talk about what is known about their relationship to Kolchin polynomials. After giving some examples and open problems, we will explain how recent work (joint with Moosa) on a Jouanolou-type theorem together with recent work (joint with Nagloo and Thieu Vo) on vector fields lead to answering a question of Hrushovski and Scanlon on ranks in differential fields.

Ruyong Feng
Academy of Mathematics and Systems Science, CAS
Title: *Parallel Telescoping*
Abstract:
Creative telescoping is a powerful technique for treating symbolic sums and integrals in an algorithmic way. This technique now has been applied to many problems from various areas of mathematics and physics. As a kind of this technique, differential creative telescoping produces from an integrand a linear differential operator satisfied by the integral. Motivated by the direct problem in parameterized differential Galois theory, we introduce the notion of parallel telescoping, which is a natural generalization of differential creative telescoping. Precisely, let $K = k(t, x_1, \cdots, x_m)$ be the field of rational functions over k and

$$\omega = \sum_{1 \le i_1 < i_2 < \cdots < i_p \le m} f_{i_1 i_2 \cdots i_p} dx_1 dx_2 \cdots dx_p$$

be a differential p-form, where the $f_{i_1 i_2 \cdots i_p}$ are elements in some differential extension field of K. Parallel telescoping produces a nonzero operator L in $k[t, \partial_t]$, called a parallel telescoper, such that

$$L(\omega) = \sum_{1 \le i_1 < i_2 < \cdots < i_p \le m} L(f_{i_1 i_2 \cdots i_p}) dx_1 dx_2 \cdots dx_p = d(\eta)$$

where η is a differential $p-1$-form and d is the exterior derivation with respect to $x_1, \cdots, x_m$. In this talk, we present a necessary and sufficient condition guaranteeing the existence of parallel telescopers for differential forms ω with $f_{i_1 i_2 \cdots i_p}$ in the set of differentially finite functions over K. When restricting $f_{i_1 i_2 \cdots i_p}$ to hyperexponential functions or algebraic functions over K, we present algorithms for deciding whether L exists or not and algorithms for computing L if it exists. This is joint work with Shaoshi Chen, Ziming Li, Michael F. Singer and Stephen M. Watt.

Emma Previato
Boston University
Title: *An arithmetically integrable system.*
Abstract:
We construct the arithmetic analog of a completely integrable system. The motion of the Euler top is described by an algebraically completely integrable Hamiltonian system. We prove that the system is Frobenius liftable to an integral manifold, whose tangent bundle is spanned by arithmetic flows generated by commuting Hamiltonians in the arithmetic sense defined by A. Buium. This is joint work with A. Buium.

Gleb Pogudin
Johannes Kepler University, Linz
Title: *New bounds for effective differential Nullstellensatz.*
Abstract:
Differential Nullstellensatz is a fundamental theorem in the algebraic theory of differential equations. It states that, if a system of algebraic differential equations does not have a solution in any extension of the coefficient field, then 1 can be expressed as a polynomial linear combination of the derivatives of the equations of the original system up to some order. Thus, finding bounds for this order is a key ingredient of algorithmic treatment of solvability of differential equations and related questions. We will present a new and improved bound for the effective version of the differential Nulstellensatz and discuss some differential-algebraic problems arising in the proof. This is joint work with Alexey Ovchinnikov and Thieu Vo Ngoc.

Charlotte Hardouin
Institut de Mathématiques de Toulouse
Title: *Parametric genericity of special functions.*
Abstract:
Among the functions of a complex variable, we distinguish the rational and algebraic functions from the transcendental functions. In the last class, we consider a new hierarchy depending on the differential dependence of the function. Starting from the holonomic or D-finite functions, that satisfy a linear differential

equation, this hierarchy defines a new level of complexity, called hypertranscendence, that corresponds to the functions that do not satisfy a polynomial relation with their derivatives. A celebrated example is the Gamma function, which is hypertranscendental by a result of Hlder.

Recently parametrized Galois theories (H.-Singer, Di Vizio-H.-Wibmer and Ovchinnikov-Wibmer) have been developed in order to understand theses questions from a geometric point of view. In this talk, I will try to expose a strategy developed in two collaborations with Dreyfus and Roques that allows to show that if a function is transcendental then it is also hypertranscendental. We mainly focus on solutions of q-difference equations and consider discrete and continuous hypertranscendence. This strategy heavily relies on a beautiful classification result for differential algebraic groups by Cassidy, that allows to reduce the question of hypertranscendence to a question of holonomy.

Victor Kac
MIT
Title: *The method of lambda brackets in the theory of integrable (bi)Hamiltonian PDE.*
Abstract:
I will introduce the notion of a Poisson vertex algebra (=differential algebra endowed with a lambda bracket) and explain how it is used to construct integrable evolution partial differential equations. Non-local and non-commutative situations will be also briefly discussed.

Omar Leon-Sanchez
McMaster University
Title: *Order bounds and Kolchin polynomials.*
Abstract:
Recently a new upper bound for the order of differential polynomials of a characteristic set of each prime component of a differential ideal has been found (for an arbitrary number of derivations m). This bound is computationally feasible (at least in the cases $m = 1, 2, 3$) and does "not" depend on the degrees of the defining equations. For instance, for the case $m = 2$, the bound is $r2^n$ where n is the number of variables and r the order of the original equations. In this talk I will present some applications around: 1) Securing an upper bound for the typical differential dimension, and 2) The definability of Kolchin polynomials in families.

Julien Roques
Institut Fourier, Universit Grenoble Alpes, France
Title: *Hypergeometric mirror maps.*
Abstract:
Mirror maps are power series which occur in mirror symmetry as the inverse for composition of power series of the form $q(z) = exp(\omega_2(z)/\omega_1(z))$, called canonical coordinates, where $\omega_1(z)$ and $\omega_2(z)$ are particular solutions of the Picard-Fuchs equation associated with certain one-parameter families of Calabi-Yau varieties. In several cases, the mirror maps have integral coef- ficients. In this talk, we will give an overview of the integrality properties of mirror maps associated to the generalized hypergeometric equations.

Wei Li
Academy of Mathematics and Systems Science, Chinese Academy of Sciences
Title: *Partial Differential Chow Forms and a Type of Partial Differential Chow varieties.*
Abstract:
In this talk, we first discuss an intersection theory for quasi-generic partial differential polynomials in partial differential algebraic geometry. Then based on the intersection theory, we define the partial differential Chow form for an irreducible differential variety whose Kolchin differential polynomial is of the form $(d+1)\binom{t+m}{m} - \binom{t+m-s}{m}$, and we show that most of the basic properties of the differential Chow form in the ordinary case are established for its partial differential counterpart. Furthermore, we prove that a special type of partial differential Chow varieties exist.

Andrei Minchenko
University of Vienna
Title: *Simple differential algebraic Lie algebras.*
Abstract:
The main purpose of the talk will be to describe the relation between differential algebraic Lie algebras in the sense of Cassidy and linearly compact Lie algebras. In particular, we will see how the classification of simple linearly compact Lie algebras can be used to deduce Cassidy's classification of simple linear differential algebraic groups.

Zoe Chatzidakis
CNRS - ENS
Title: *Definable Galois theory in difference fields.*
Abstract:
I will speak about the group of automorphisms of solutions (in a suffiuciently rich difference field) of difference equations which are internal to the fixed field. (The typical example of such equations is linear difference equations.).

In particular, we show that if the field K over which the equations are defined is algebraically closed, with fixed subfield pseudo-finite, then the set of solutions of our system is strongly related with the set of solutions of an equation of the form f(x)=gx and x in X, where g is an element of an algebraic group G acting on a algebraic set X, and g is in G(K).

I will also discuss what happens in the general case. If time permits, I will mention some applications. (Joint work with Ehud Hrushovski)

Georg Regensburger
Johannes Kepler University Linz
Title: *The fundamental theorem of calculus in differential algebra.*
Abstract:
For incorporating the fundamental theorem of calculus in differential algebras, we consider a linear right-inverse of the derivation. In these algebras, we automatically have an induced evaluation operation corresponding to the second part of the fundamental theorem. This setting generalizes the notion of integro-differential algebras where in addition the evaluation has to be multiplicative.

We construct the corresponding algebra of linear operators as a quotient of a tensor algebra. Based on a completion process for tensor reduction systems, we find all linear consequences (like integration by parts) of the defining relations. This, in turn, enables us to determine normal forms for these generalized integro-differential operators. Normal are needed for effective computations and are implemented as a Mathematica package. We also illustrate how analogs of the Taylor formula or variation of constants can be proven in this operator framework. Joint work with Clemens G. Raab.

Ekaterina Shemyakova
SUNY New Paltz
Title: *Differential operators on the superline, Berezinians and Darboux transformations.*
Abstract:
We consider nondegenerate differential operators on the superline (i.e., a supermanifold of dimension 1—1). We show how every such operator can be expressed via certain "super Wronskians" (particular Berezinians). This is applied to classification of Darboux transformations (DT). Namely, we show that every DT on the superline corresponds to an invariant subspace of the source operator and upon a choice of a basis is given by a super-Wronskian formula. These formulas are non-trivial because Berezinians are fractions, not polynomials. (On the way, we obtain for Berezinians some analogs of classical row and column expansions, probably new.) We also discuss a generalization of the above to differential operators acting on the algebra of densities on the superline.

See `https://arxiv.org/abs/1605.07286` and `https://arxiv.org/abs/1505.05194`

Michael Wibmer
University of Pennsylvania
Title: *Torsors for difference algebraic groups.*
Abstract:
We introduce a cohomology set for groups defined by algebraic difference equations and show that it classifies torsors under the group action. This has applications to the parameterized Picard-Vessiot theory.

Joel Nagloo
Bronx Community College CUNY
Title: *The third Painlevé transcendents revisited.*
Abstract:
In this talk I will revisit the problem of existence of algebraic relations over $\mathbb{C}(t)$ between solutions (and derivatives) of the generic third Painlevé equation. In particular, I will discuss how one can reduce that problem to studying the Riccati subvarieties.

Taylor Dupuy
University of Vermont
Title: *Arithmetic Deformation Classes in the Sense of Buium.*
Abstract:
In a 1999 paper, Faltings poses a simple property that an "Arithmetic Kodaira-Spencer class" must satisfy. He then shows that such a class cannot exist: `https://goo.gl/uXygB1`

If one modifies this definition and uses Buium's wittferential equations, the so-called Deligne-Illusie class naturally appears and one a construction which satisfies a slightly modified version of Falting's axiom. We will explain all of this in our talk.