BRONX COMMUNITY COLLEGE
of the City University of New York
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

CSI30   **Midterm Exam Sample**

Instructions

Every problem is worth 10 points. Solve all problems and mark your answers clearly. Simplify your answers whenever possible. Show all work, using additional paper if needed. Remember to write your name on all paperwork you hand in.

1. Construct a truth table for these compound propositions:
   a. $(p \oplus q) \oplus r$
   b. $(p \rightarrow q) \rightarrow r$

2. Evaluate each of these expressions:
   a. `1111 0000` $\oplus$ `1010 1010`
   b. `1111 0000` $\wedge$ `1010 1010`

3. Label the following compound propositions as tautology, inconsistency, or contingency:
   a. $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
   b. $p \rightarrow (p \wedge q)$
   c. $p \wedge \neg p$
   d. $p \vee (p \wedge q)$

4. Express the following English sentence as a compound proposition (Be sure to give definitions of any propositional variables you use):
   Jack will not fail the class only if he gets 80 on the final exam and completes a special project.

5. Suppose that the domain of the propositional function $P(x)$ consists only of the integers 0, 1, and 2. Expres these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions:
   a. $\forall x \neg P(x)$
   b. $\exists x P(x)$

6. Rewrite the following statements as equivalent statements, but with all quantifiers to the left of any negation symbols:

$\neg\forall x\exists y P(x,y)$

$\neg\exists x\forall y Q(x,y)$

7. Give the truth values of the following statements, and explain your answers:

$\forall x\exists y(x=2y)$ where the universe consists of all integers $\mathbb{Z}=\{...,-2,-1,0,1,2,...\}$.∎

$\forall x\exists y(y=\frac{1}{x})$ where the universe consists of all real numbers $\mathbb{R}$.

8. Express the following sentence using predicates, quantifiers, and logical connectives (Be sure to give the meaning of any predicates you use. Also, indicate the domain of the predicates, i.e., the universe):

If anyone in the platoon acts carelessly, then everyone in the platoon is in danger.

9. Let $X=\{a,b,c,d,e,f\}$, $Y=\{a,e,i,o,u\}$. What are $X\cap Y$, $X-Y$, $Y-X$, and $X\cup Y$?

10. Show that the intersection of set A and set B is exactly equal to the difference of the complement of set B and set A, i.e., show that $A\cap B=A-\overline{B}$. (You could do this by drawing two appropriate Venn diagrams, or by using the definitions of the set operations (which elements are in the difference of two sets, for example) and finding equivalent logical formulas to show the sets are the same.)