CSI33 Data Structures

Department of Mathematics and Computer Science
Bronx Community College

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Outline

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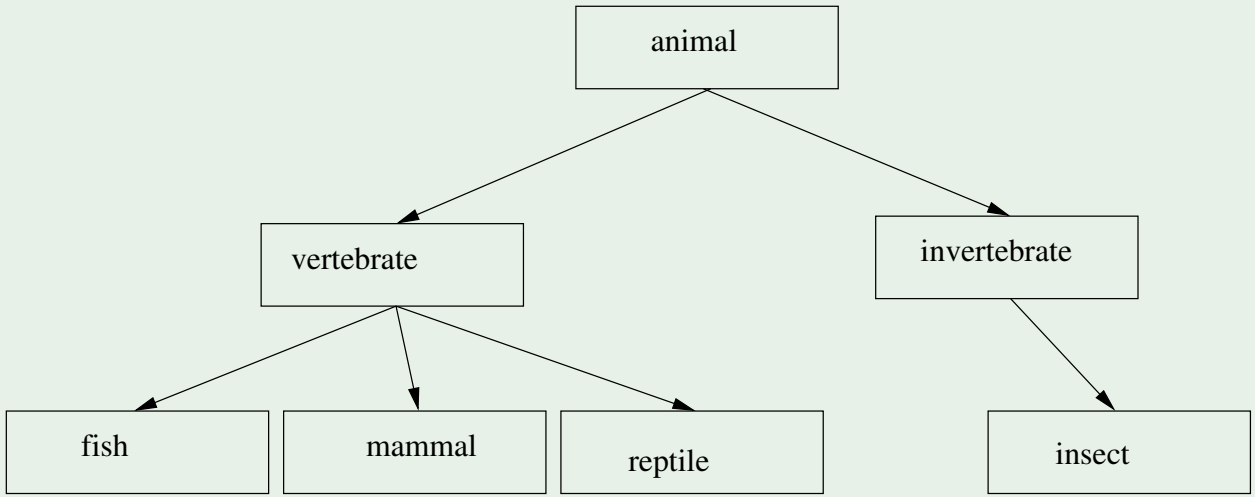
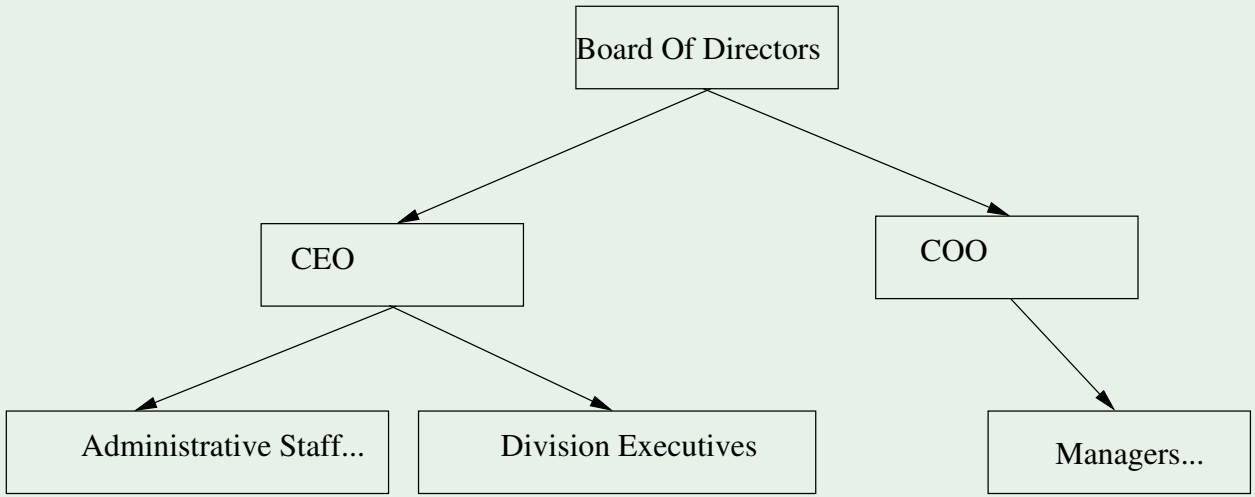
Uses Of Trees

Taxonomies



Uses Of Trees

Hierarchies



Uses Of Trees

Efficient Containers of Sequential Data

Each node represents a single data item of a collection organized, for efficient access, into a tree:

- Binary Search Trees
- Heaps
- Priority Queues
- B-Trees, Quad Trees, etc.

Definitions

Definitions I

- A tree consists of nodes connected by edges.
- A node can have zero or more child nodes. A child is connected to its parent node by a single edge.
- Children with the same parent are called siblings.
- A tree with no nodes or edges is an empty tree.
- A nonempty tree will have one special node with no parent called the root node.
- A node with no children is called a leaf.

Definitions

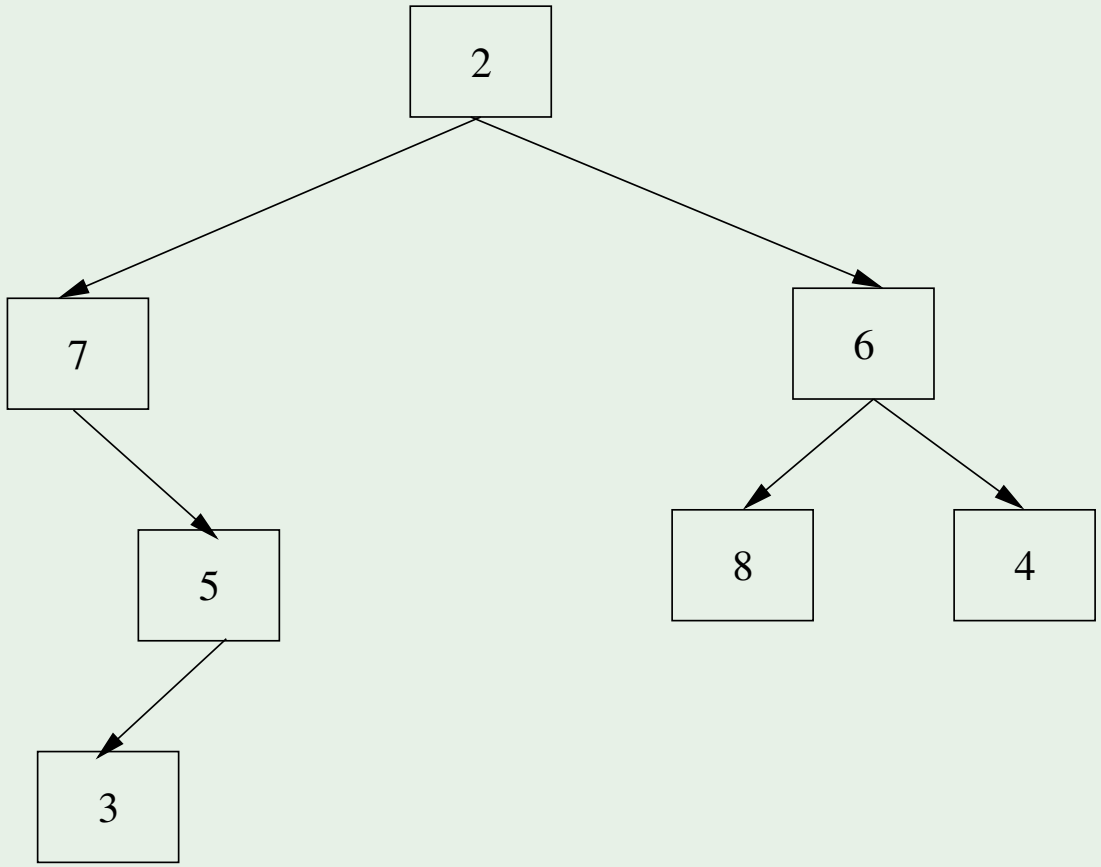
Definitions II

- All nodes are connected to the root by a path of edges.
- The depth of a node is the length of its path to the root. The root has depth zero.
- A level of a tree is a set of nodes which have the same depth.
- The descendants of a node are nodes whose paths include that node.
- The ancestors of a node are nodes on its path to the root.

Binary Trees

Binary Trees

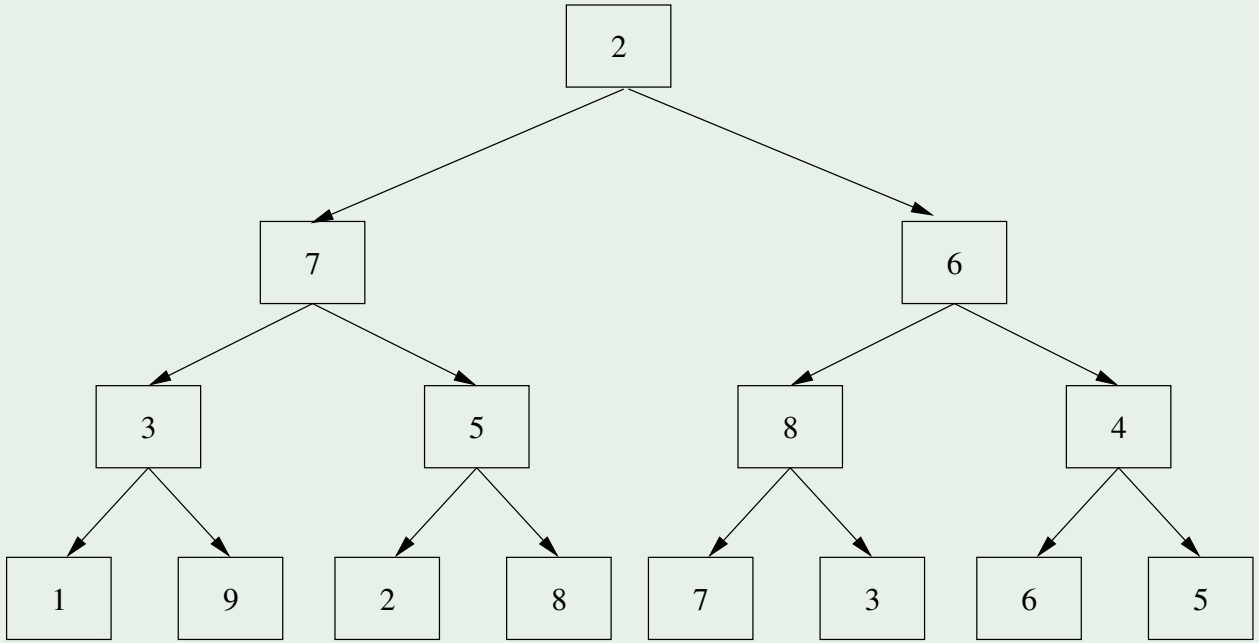
- A tree whose nodes have at most two children is a binary tree.



Binary Trees

Full Binary Trees

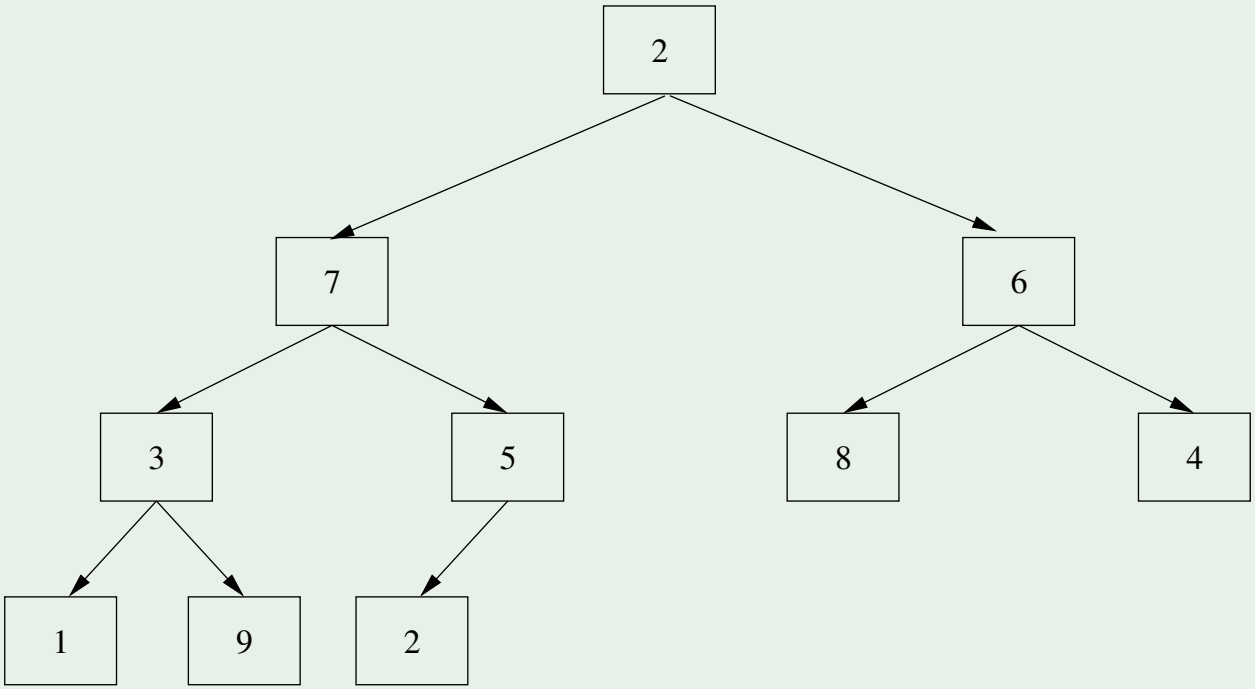
- A full binary tree is one whose levels have every position filled.



Binary Trees

Complete Binary Trees

- A complete binary tree has every level filled except the bottom, which is filled from left to right.



Binary Tree Traversal

Binary Trees are Recursive Data Structures

A binary tree can be defined as either

- An empty binary tree (base case!), or
- a node having two binary trees as attributes, a left subtree and a right subtree (recursive case).
- Using this recursive definition, recursive algorithms can be used to process the nodes of a binary tree.

Binary Tree Traversal

Preorder

```
def traverse(tree):
   if tree is not empty:
      process data at tree's root
      traverse(tree's left subtree)
      traverse(tree's right subtree)
```

Binary Tree Traversal

Inorder

```
def traverse(tree):
   if tree is not empty:
      traverse(tree's left subtree)
      process data at tree's root
      traverse(tree's right subtree)
```

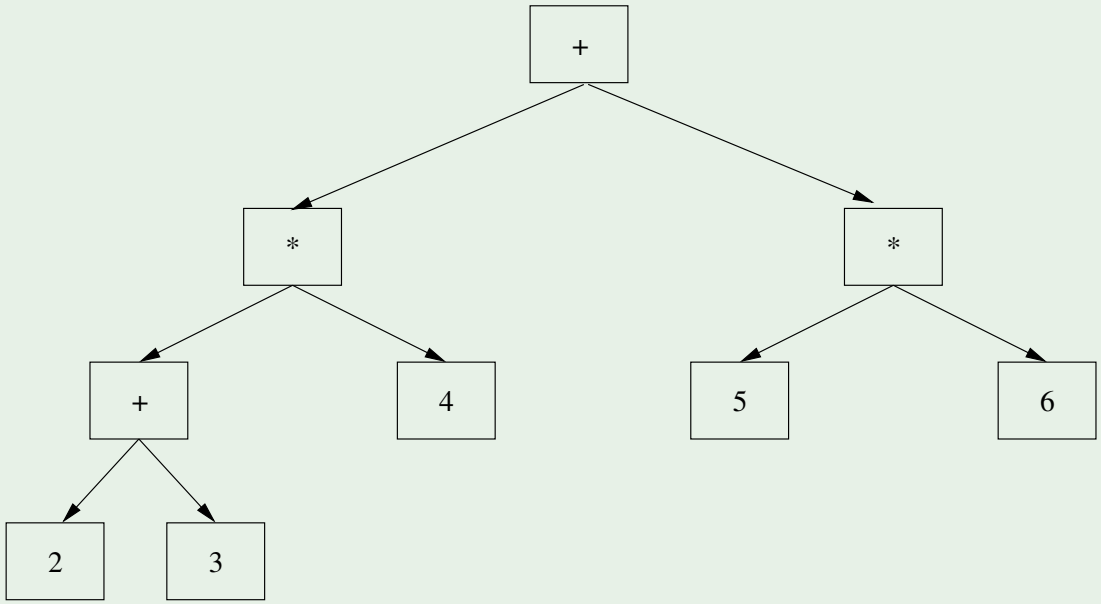
Binary Tree Traversal

Postorder

```
def traverse(tree):
   if tree is not empty:
      traverse(tree's left subtree)
      traverse(tree's right subtree)
      process data at tree's root
```

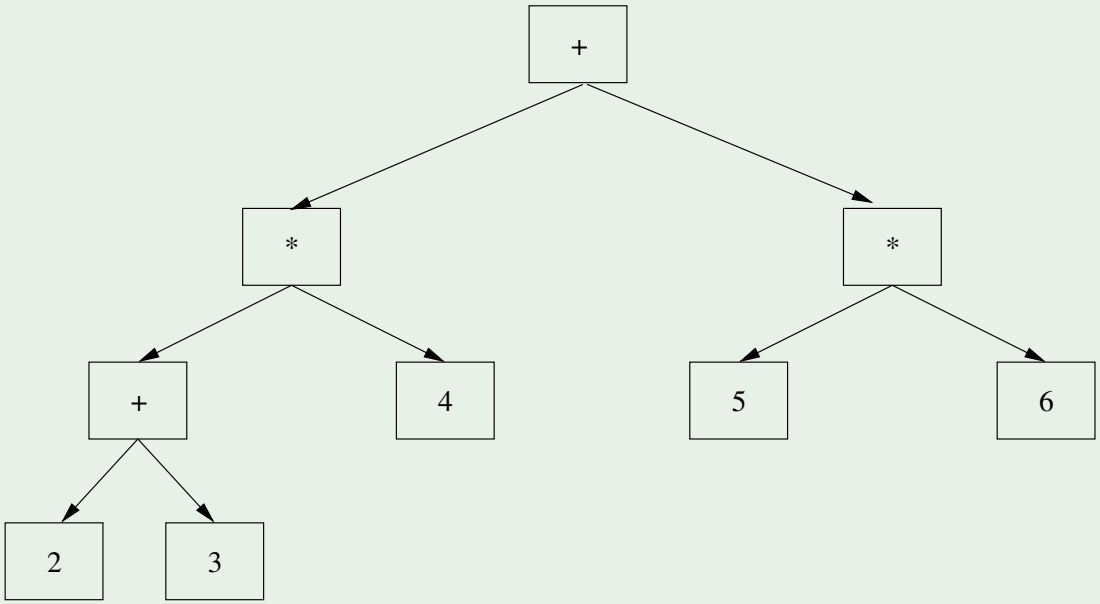
Binary Tree Representation Of An Expression

Abstract Syntax Tree



Binary Tree Representation Of An Expression

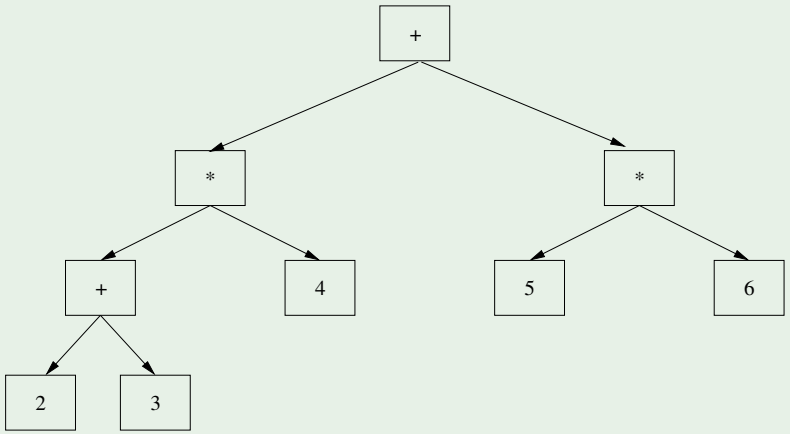
Different Traversals Give The Different Notations



- Printing in preorder gives Prefix Notation.
- Printing inorder gives Infix Notation.
- Printing in Postorder gives Postfix Notation.

Binary Tree Representation Of An Expression

Evaluation




```
def evaluateTree(tree):
  if tree's root is an operand:
     return root data
  else:  # root contains an operator
     leftValue = evaluateTree(tree's left subtree)
     rightValue = evaluateTree(tree's right subtree)
     result = operator(leftValue, rightValue)
     return result
```

Linked Representation Of A Binary Tree in C++

Use A TreeNode Class

- A binary tree has a single instance variable, `_root`, a pointer to a `TreeNode`.
- A nonempty binary tree is represented with `_root` as a pointer to an actual `TreeNode` object.
- An empty binary tree has no root. We represent this by using a value of NULL for the instance variable `_root`. as a pointer to a `TreeNode`.
- The TreeNode class is defined recursively.

Linked Representation Of A Binary Tree in C++

The TreeNode class is defined recursively.

- The `_left` and `_right` instance variables are pointers to `TreeNode` objects, if there are left or right subtrees, or `NULL` if those subtrees are empty.
- The `_item` instance variable is the data item stored in the TreeNode.

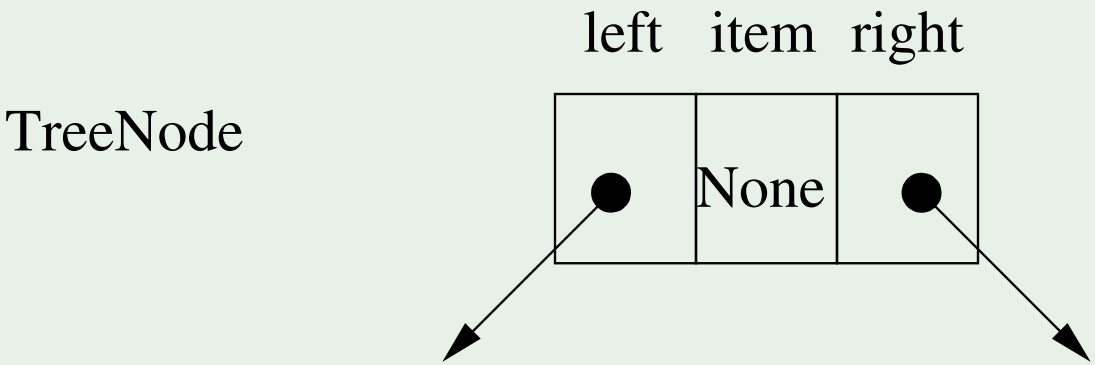
Linked Representation Of A Binary Tree in C++

(Simplified) Definition Of A TreeNode Class in C++

```
class TreeNode {
public:
   TreeNode(int item, TreeNode* left = NULL,
TreeNode* right = NULL);
   ~TreeNode(); // destructor
   int _item; // value
   TreeNode* _left; // link to left subtree
   TreeNode* _right; // link to right subtree
};
```

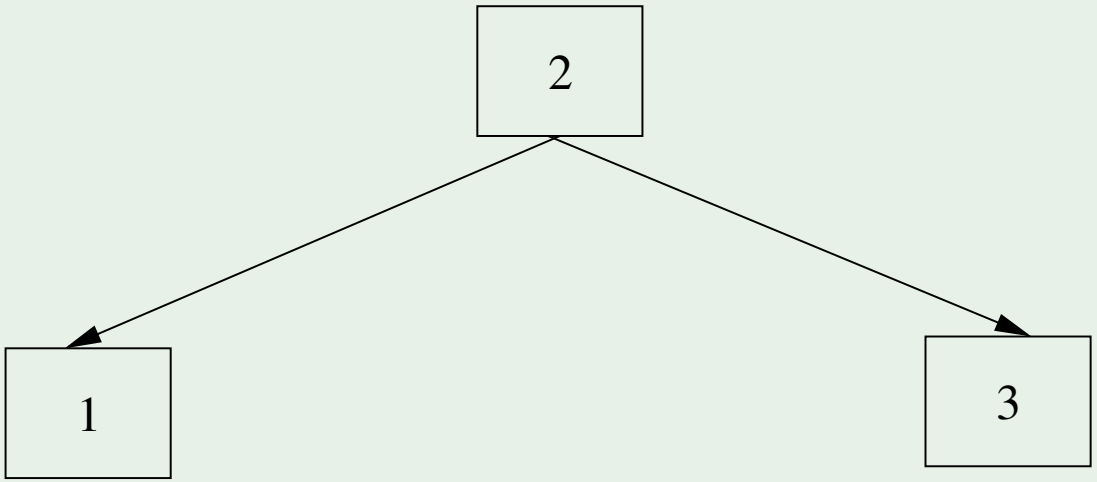
Linked Representation Of A Binary Tree in C++

C++ Examples Using The TreeNode Class



Linked Representation Of A Binary Tree in C++

Example 1: Abstract View



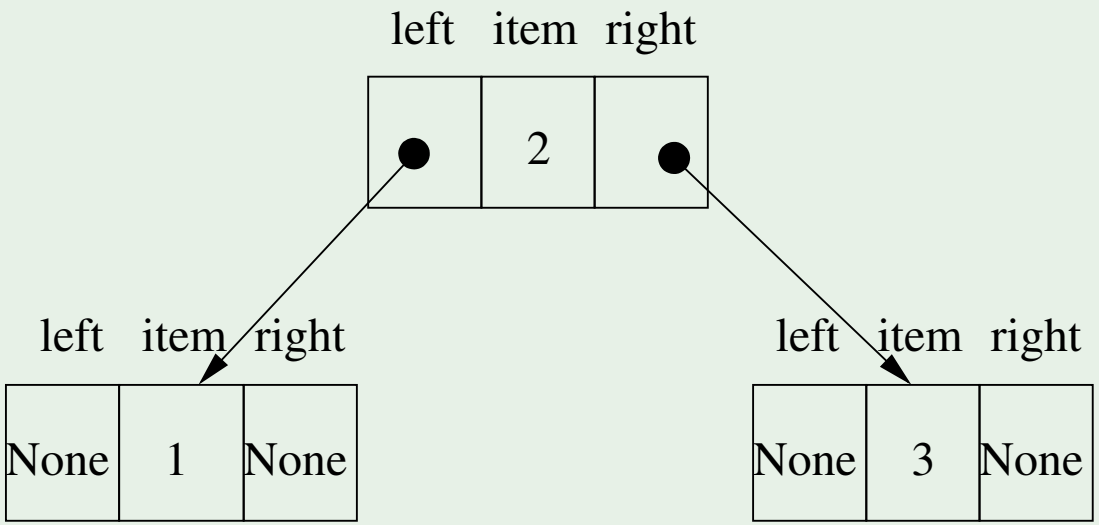
Linked Representation Of A Binary Tree in C++

Example 1: C++ Implementation

```
TreeNode *_left = new TreeNode(1);
TreeNode *_right = new TreeNode(3);
TreeNode *_root = new TreeNode(2, _left, _right);
```

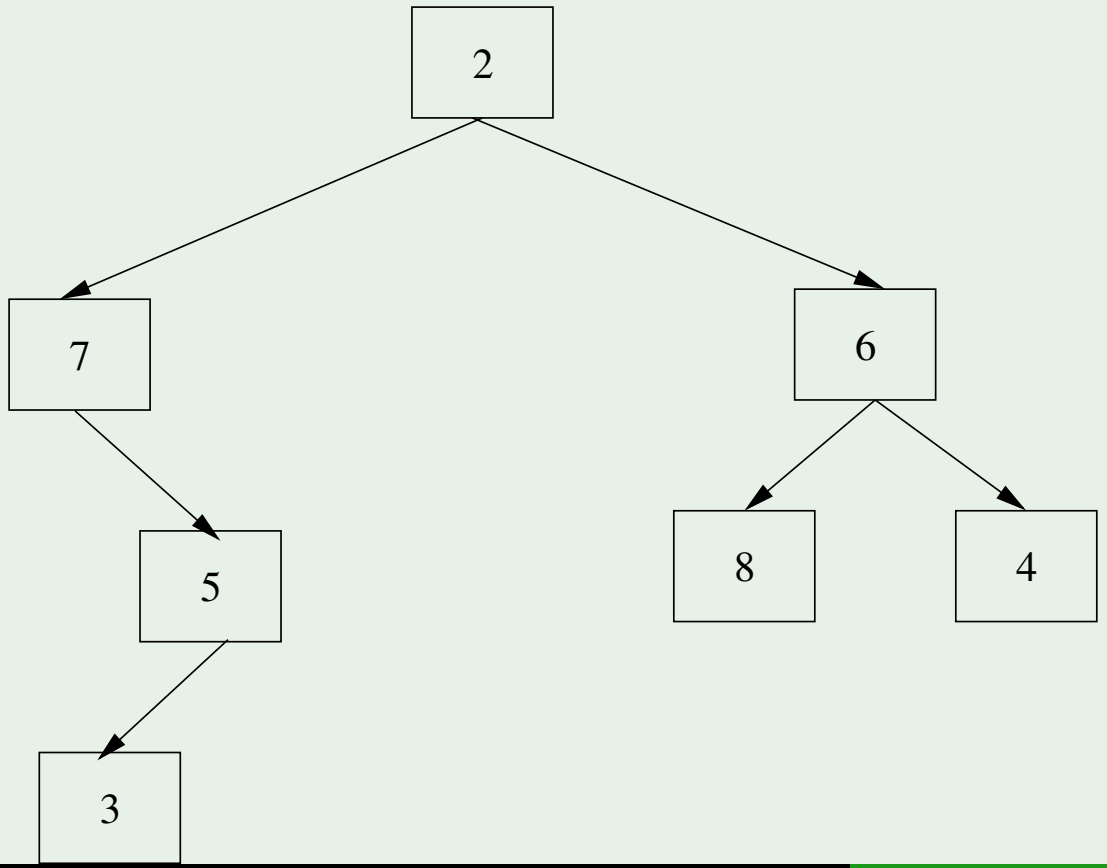
Linked Representation Of A Binary Tree in C++

Example 1: C++ Memory Model



Linked Representation Of A Binary Tree in C++

Example 2: Abstract View



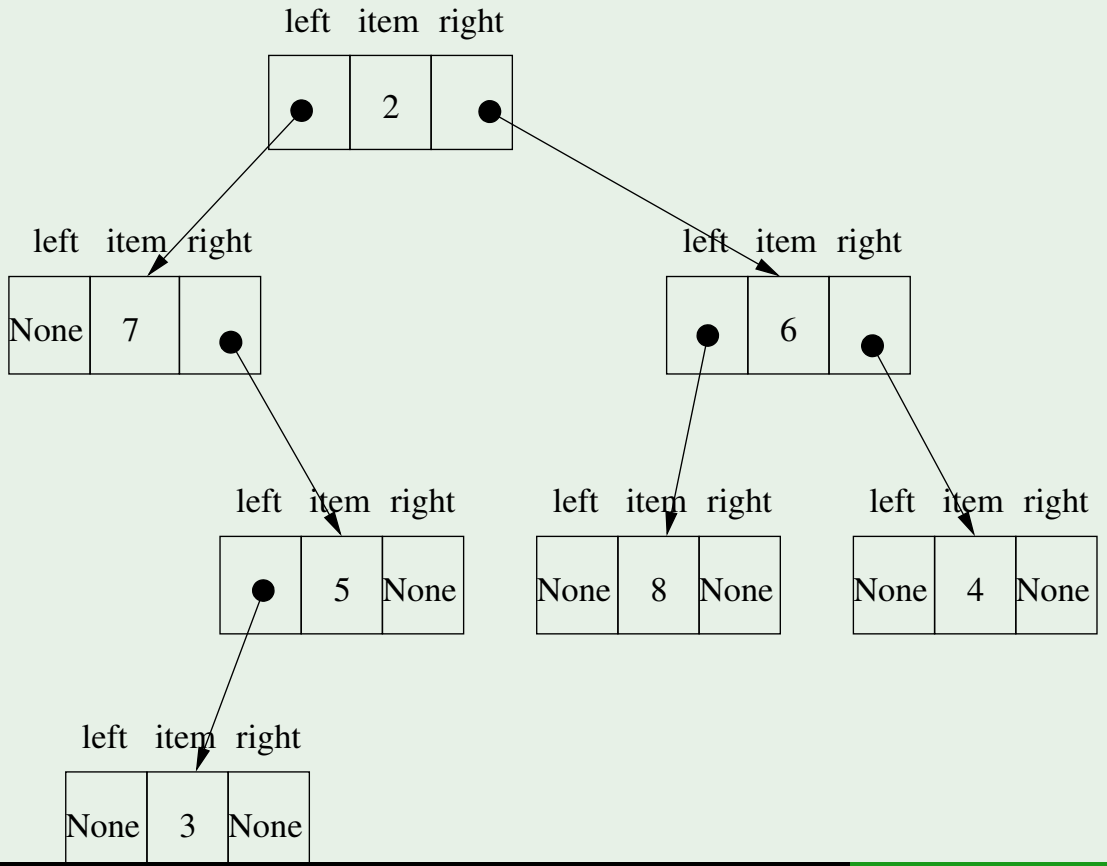
Linked Representation Of A Binary Tree in C++

Example 2: C++ Implementation

```
_root = new TreeNode(2,
          new TreeNode(7,
             NULL,
             new TreeNode(5, new TreeNode(3))),
          new TreeNode(6, new TreeNode(8), new
TreeNode(4)));
```

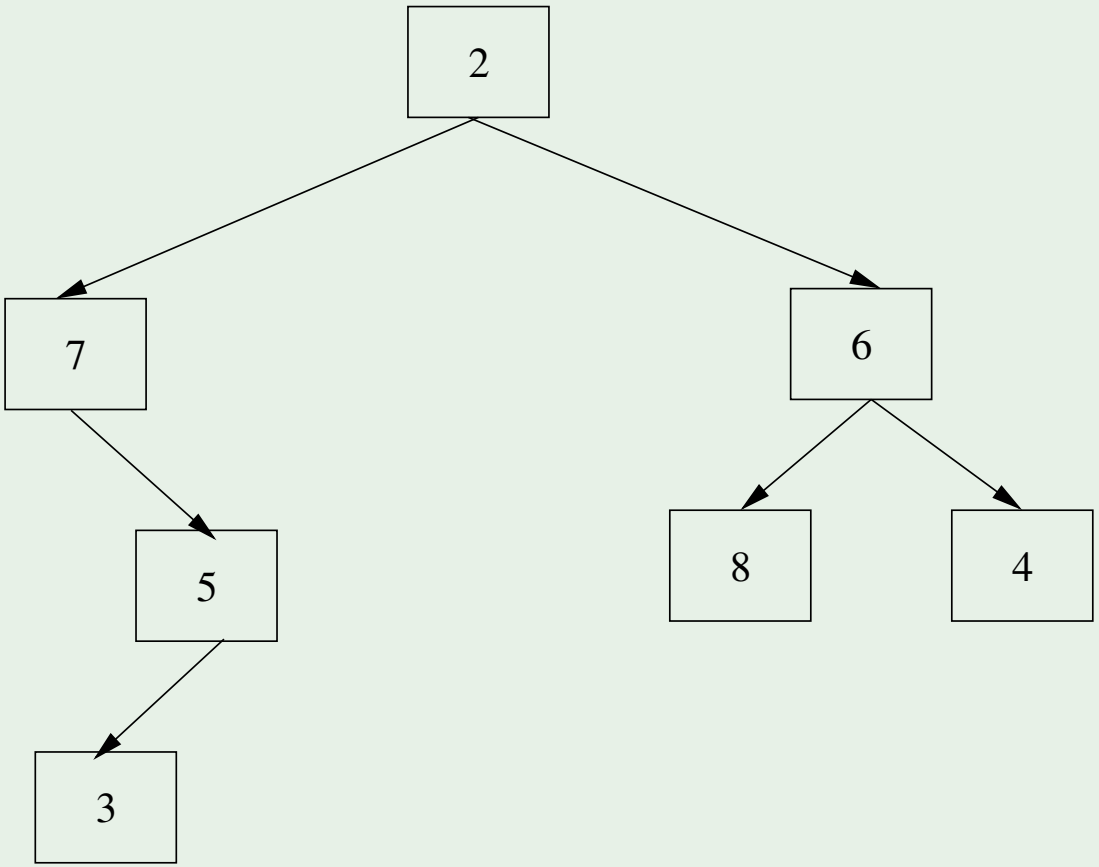
Linked Representation Of A Binary Tree in C++

Example 2: C++ Memory Model



Array Representation of Binary Trees

Abstract View



Array Representation of Binary Trees

Implementation

2	7	6	NULL	5	8	4	NULL	NULL	3
1	2	3	4	5	6	7	8	9	10

```
def left_child(i):
   return 2 * i
def right_child(i):
   return 2 * i + 1
def parent(i):
   return i // 2
```