CSI33 Data Structures

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Outline

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Measuring Complexity (Running Time) Of Recursive Algorithms

Comparison With Iterative (Looping) Algorithms

- Any iterative algorithm can be transformed into a recursive one.
- Different strategies lead to different running times. (The recursive power example is more efficient than the naive loop version.)
- To measure efficiency, you must count recursive calls and the depth of the call stack.
- You must also consider the size of the data parameters that are passed in recursive calls.

The Fibonacci Sequence

The Fibonacci Sequence

The Fibonacci Sequence is obtained by beginning with the pair of numbers $1, 1$ and continuing indefinitely by adding the last two numbers to give the next number in the sequence, giving 1, 1, 2, 3, 5, 8, 13 and so on.

The Fibonacci Sequence

The nth Fibonacci Number: Loop Version

```
def loopFib(n):
   curr = 1
   prev = 1
   for i in range(n - 2):
      curr, prev = curr + prev, curr
   return curr
```

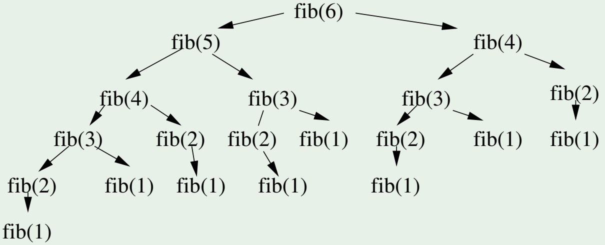
The Fibonacci Sequence

The nth Fibonacci Number: Recursive Version

```
def recFib(n):
    if n < 3:
        return 1
    else:
        return recFib(n - 1) + recFib(n - 2)
```

The Fibonacci Sequence

Analysis



The Fibonacci Sequence

Analysis

To calculate `fib(6)` is very wasteful:

- `fib(4)` is calculated 2 times
- `fib(3)` is calculated 3 times
- `fib(2)` is calculated 5 times
- `fib(1)` is calculated 8 times
  To calculate `fib(n)` requires $fib(n) - 1$ steps, so the running time is $\Theta(fib(n))$, which is $\Theta(2^n)$), or exponential in n.

The Fibonacci Sequence

The nth Fibonacci Number: Improved Recursive Version

```
def newFib(n):
  return newFib2(1, 1, 0, n)
def newFib2(curr, prev, i, n):
  if i == n - 2:
    return curr
  else:
    return newFib2(curr + prev,curr, i + 1, n)
```

The Fibonacci Sequence

Analysis

To calculate `fib(n)` now requires $n - 2$ recursive calls, so the running time is $\Theta(n)$, which is big improvement.

The Fibonacci Sequence

How To Make An Iterative Function Recursive

- Write a function that calls a helper function with parameters for all local variables and parameters from the loop version.
- Pass the initial values from the loop version in this function call.
- The helper function will be recursive:
- The base case will be the negation of the loop condition.
- The recursive call will change the parameters to match one iteration of the loop version.

Selection Sort

Selection Sort

```
def SelectionSort(lst):
    n = len(lst)
    for i in range(n-1):
        pos = i
        for j in range(i+1, n):
            if lst[j] < lst[pos]:
                pos = j
        lst[i], lst[pos] = lst[pos], lst[i]
```

Selection Sort

Selection Sort Analysis

- Inner loop runs n times
- First time it compares n items, then $n-1$, etc.
- Total comparisons $= n + (n-1) + (n-2) + \ldots + 1 = \frac{n(n+1)}{2}$
- Running time is $\Theta(n^2)$

Recursive Design: Mergesort

Mergesort Pseudocode

```
Algorithm: mergeSort nums
   split nums into two halves (nums1, nums2)
   sort nums1 (the first half)
   sort nums2 (the second half)
   merge nums1 and nums2 back into nums
```

Recursive Design: Mergesort

Merge Pseudocode

```
Algorithm: merge sorted lists (nums1 and nums2) into
nums:
   while both nums1 and nums2 have more items:
      if top of nums1 is smaller:
         copy it into current spot in nums
      else (top of nums2 is smaller):
         copy it into current spot in nums
   copy remaining items from nums1 or nums2 to nums
```

Recursive Design: Mergesort

Recursive mergeSort

```
if len(nums) > 1:
   split nums into two halves (nums1, nums2)
   mergeSort nums1 (the first half)
   mergeSort nums2 (the second half)
   merge nums1 and nums2 back into nums
```

Analyzing Mergesort

Running Time of merge

- Each item gets moved exactly once back into `nums`
- Running time is $\Theta(n)$, where n is the size of `nums`

Running Time of mergeSort

- The call stack gets as deep as $\log_2(n)$, where n is the size of `nums`
- At each stage, `mergeSort` is called twice, but for each call, the argument list is half the size as before.
- For $\log_2(n)$ stages, each of the n items is moved once per stage.
- The running time is the product, which is $\Theta(n \log n)$

Analyzing Mergesort

Running Time of mergeSort