

# CSI33 DATA STRUCTURES

Department of Mathematics and Computer Science  
Bronx Community College

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# OUTLINE

- 1 CHAPTER 5: STACKS AND QUEUES
  - Stacks



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# THE STACK ADT

## A CONTAINER CLASS FOR LAST-IN-FIRST-OUT ACCESS

A **stack** is a list-like container with access restricted to one end of the list (the **top** of the stack). You can

- **push** an item onto the stack
- **pop** an item off the stack (precondition: stack is not empty— $\text{size} > 0$ )
- Inspect the **top** position (precondition: stack is not empty— $\text{size} > 0$ )
- Obtain the current **size** of the stack.



# SIMPLE STACK APPLICATIONS

## BALANCED PARENTHESES

```
def parensBalance2(s):
    stack = Stack()
    for ch in s:
        if ch in "([{": " push an opening marker "
            stack.push(ch)
        elif ch in ")]}": " match closing "
            if stack.size() < 1: " no pending open "
                return False
            else:
                opener = stack.pop()
                if opener+ch not in ["()", "[]", "{}"]:
                    return False " not a matching pair"
    return stack.size() == 0 " everything matched?"
```



# SIMPLE STACK APPLICATIONS

## BALANCED PARENTHESES

{ [ 2 \* ( 7 - 4 ) + 2 ] + 3 } \* 4



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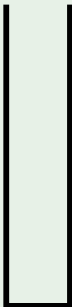
{ [ 2 \* ( 7 - 4 ) + 2 ] + 3 } \* 4

{

# SIMPLE STACK APPLICATIONS

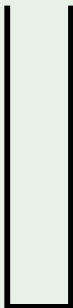
## BALANCED PARENTHESES

{ [ 2 \* ( 7 - 4 ) + 2 ] + 3 } \* 4



## SIMPLE STACK APPLICATIONS

## BALANCED PARENTHESES

$$\{ [ 2 * ( 7 - 4 ) + 2 ] + 3 \} * 4$$


# IMPLEMENTING STACKS

## A PYTHON LIST AS CONCRETE REPRESENTATION

In Python the natural implementation of a stack is with a list.

```
class Stack(object):
    def __init__(self):
        self.items = []
    def push(self, item):
        self.items.append(item)
    def pop(self):
        return self.items.pop()
    def top(self):
        return self.items[-1]
    def size(self):
        return len(self.items)
```





# AN APPLICATION: EXPRESSION MANIPULATION

## NOTATION FOR OPERATIONS

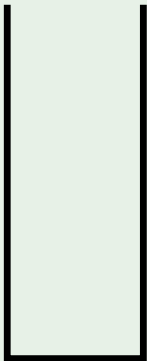
- **infix**:  $(2 + 3) * 4$
- **prefix**:  $* + 2 3 4$
- **postfix**:  $2 3 + 4 *$



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

3

## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

4

3



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

5

4

3



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

9

3



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

27



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

2

27





## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

25



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

3

25



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

6

3

25



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

18

25



## AN APPLICATION: EXPRESSION MANIPULATION

## EVALUATING A POSTFIX EXPRESSION

3 4 5 + \* 2 - 3 6 \* +

43



# THE CALL STACK

## FUNCTION CALLS CAN BE NESTED

- function A calls function B
- function B returns
- function A continues



# THE CALL STACK

## ACTIVATION RECORDS

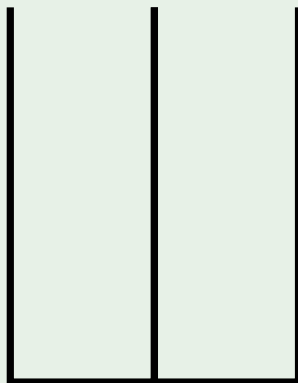
- Function A is running, and calls function B.
- The local variables of function A, their current values, and where function B should return to are put into an **activation record**.
- The activation record is pushed onto the **call stack** which has been allocated for the program that is running.
- When function B returns, this record is popped off the call stack and used to continue running the program.



## THE CALL STACK

## EXAMPLE

```
def A(x, y):  
    1:  x2 = B(x)  
    2:  y2 = B(y)  
    3:  z = x2 + y2  
    4:  return z  
def B(n):  'squares n '  
    5:  n2 = n * n  
    6:  return n2  
def main():  
    7:  a = 3  
    8:  b = 4  
    9:  c = A(a, b)  
    10: print(c)  
    11: return
```



locals

return

Call Stack

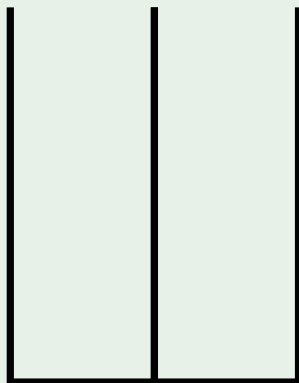




## THE CALL STACK

## EXAMPLE

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```



locals      return

Call Stack

a = 3



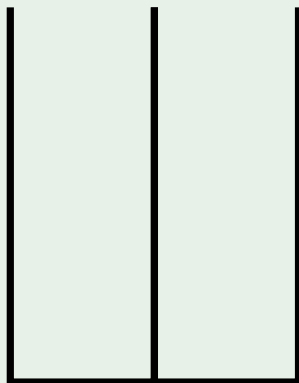
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```



locals      return

Call Stack

a = 3, b = 4



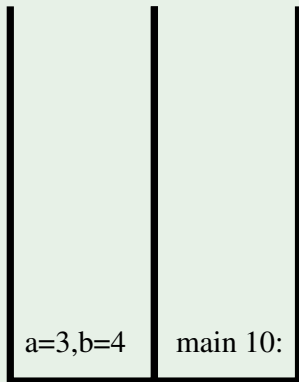
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locals      return

Call Stack

x = 3, y = 4



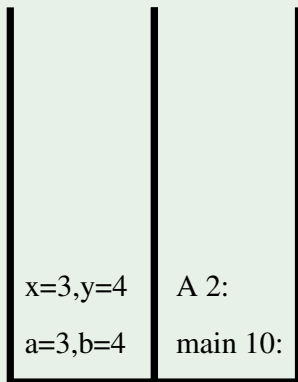
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```



locals

return

Call Stack

 $n = 3$ 

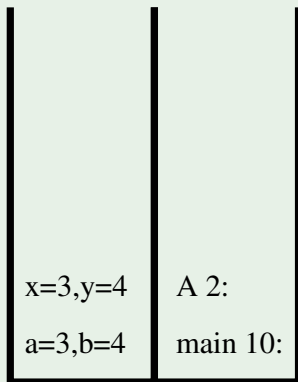
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```



locals

return

Call Stack

n = 3. n2 = 9



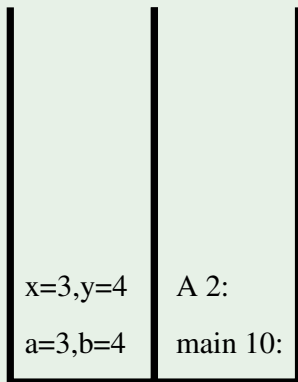
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locals

return

Call Stack

n = 3, n2 = 9



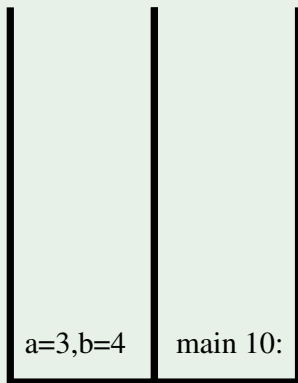
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locals      return

Call Stack

x = 3, y = 4, x2 = 9



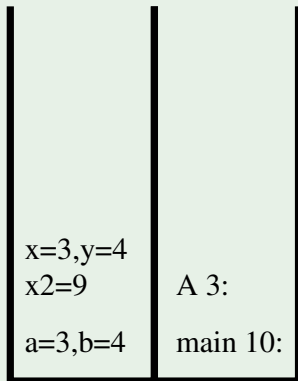
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```



locals

return

Call Stack

n = 4





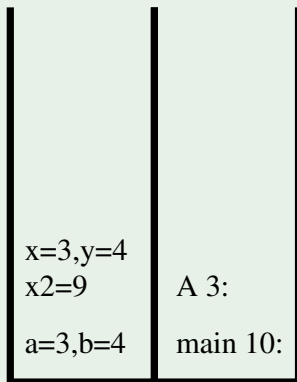
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```



locals

return

Call Stack

n = 4. n2 = 16



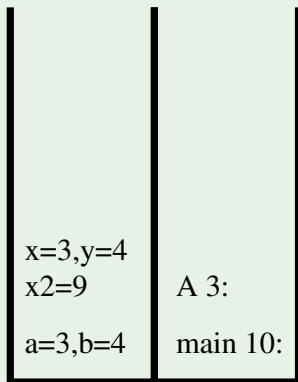
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locals

return

Call Stack

n = 4, n2 = 16



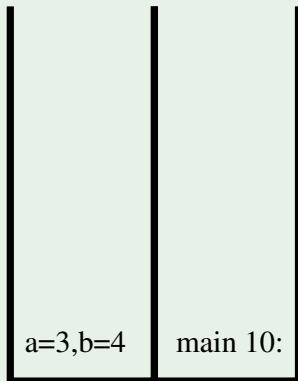
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```



locals      return  
Call Stack

x=3, y=4, x2=9, y2=16



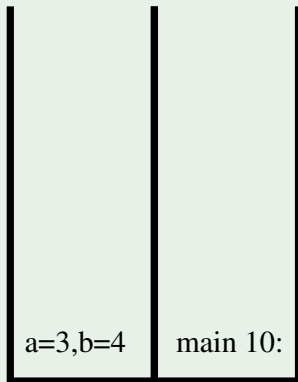
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```



locals      return

Call Stack

x=3, y=4, x2=9, y2=16, z=25



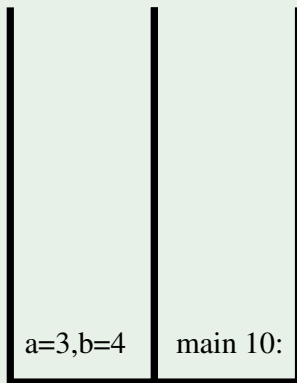
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locals      return

Call Stack

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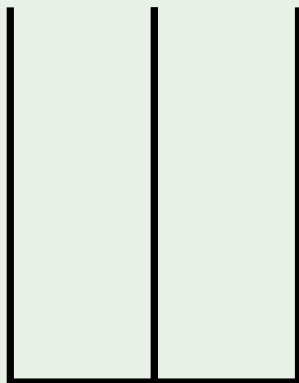
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locals      return

Call Stack

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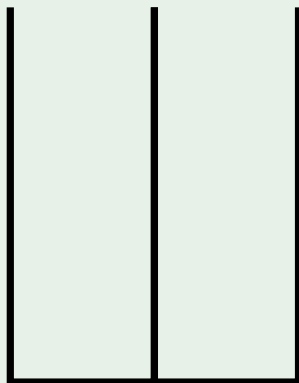
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locals      return

Call Stack

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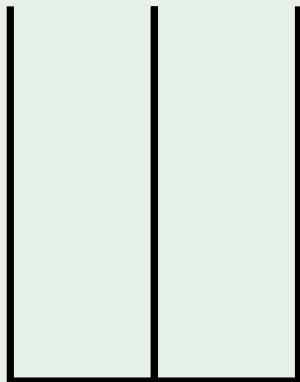
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```



locals      return

Call Stack

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