Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points for a total of 18.

(1) Let $A = \{5, 7\}$ and $B = \{0, 1, 2, 3, 4\}$. Suppose $f(5) = 3$ and $f(7) = 2$.

(a) Is f a function from A to B? Explain.

(b) What is the range of f? Write your answer in the correct set notation.

(2) Let g be the function that sends every word to its first letter. So

$$g(\text{dog}) = \text{d}$$

for example. The domain of g is the set of English words and its codomain is the set of 26 letters.

(a) Is g onto? Explain.

(b) Is g one-to-one? Explain.

(3) Let $S = \{1, 2, 3\}$ and $T = \{4, 5, 6\}$. Suppose $f : S \to T$ is defined by $f(x) = 7 - x$.

(a) What is $f(1)$?

(b) Draw the picture of f, as we did in class, showing the domain, codomain and connecting arrows.

(c) Explain why f is a one-to-one correspondence.

(d) Then f has an inverse. Find $f^{-1}(5)$.

(4) Let $p : \mathbb{R} \to \mathbb{R}$ be given by the formula $p(x) = 4x - 3$.

(a) Find: $p(0)$

(b) Find: $p(-5)$

(c) Find a number x so that: $p(x) = 2$

(5) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 3x + 5$ and let $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = 4 - x$. Answer these questions involving composition of functions.

(a) Show that $(f \circ g)(1) = 14$
(Hint: First find $g(1)$ then find f of that number. Recall $(f \circ g)(x)$ means $f(g(x))$.)

(b) Find: $(g \circ f)(1)$

(c) Find: $(f \circ f)(0)$

(d) Find a formula for: $(f \circ f)(x)$

(6) The floor function $\lfloor x \rfloor$ is a useful function from $\mathbb{R}$ to $\mathbb{Z}$ that sends a real number x to the closest integer that is less than or equal to x. The ceiling function $\lceil x \rceil$ is a similar function from $\mathbb{R}$ to $\mathbb{Z}$ that sends x to the closest integer that is greater than or equal to x. Compute

(a) $\lfloor 17.6 \rfloor$

(b) $\lceil 19 \rceil$

(c) $\lfloor -8.93 \rfloor$

(d) $\lceil -3/4 \rceil$

(The answers to parts (a), (b), (c) and (d) must be integers.)

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.

- Check if you get the right answer for a similar odd-numbered question in the text-book (answers at the back of the book).

- Ask me about it after class.

- Come to my office hours: Tue 3 - 4, Wed 3 - 4 in CP 317.

- Go to the Math Tutorial Lab in-person in CP 303 or online.