Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points for a total of 30.

(1) Let $P(x)$ say " $x^2 = 9$ " and let $Q(x)$ say " $x \geqslant 1$ ". Let the domain of discourse be the integers. Find the truth values of these statements and explain your answers.

    (a) $\forall x \, Q(x)$

    (b) $\exists x \, (P(x) \wedge Q(x))$

    (c) $\forall x \, (P(x) \vee \neg P(x))$

(2) Translate these sentences into logical expressions using quantifiers and propositional functions. Make sure you say what the domain of discourse is.

    (a) All fish can swim.

    (b) Someone in this class speaks Japanese.

    (c) Everyone in this class speaks Japanese or does not enjoy logic questions.

(3) For each part of Question (2), write the negation of each statement with propositions and quantifiers so that no negation is to the left of a quantifier. Then express the negation in simple English.

(4) Decide the truth values of these expressions, with domain the real numbers, explaining your answers:

    (a) $\forall x \forall y \, (2x + 4y = 2(x + y + y))$

    (b) $\exists x \exists y \, (x^2 + y^2 + 8 = 0)$

    (c) $\forall x \forall y \exists z \, (x^2 + y^2 = z^2)$

(5) Let $F(x, y)$ say "x can fool y". The domain is all people in the world. Use quantifiers to express each of these:

    (a) Everybody can fool Ben.

    (b) Maria can fool everybody.

    (c) Everybody can fool somebody.

    (d) There is no one who can fool everybody.

    (e) Everyone can be fooled by somebody.

    (f) No one can fool both Ben and Jerry.

(6) Express this sentence in logic and give its negation in logic and in English (make sure that no negation is to the left of a quantifier). Use $T(x, y)$ to say "x follows y on Twitter".

"There is someone that everyone follows on Twitter".

(7) Suppose these two statements are true:

- Today is Sunday.
- If it is Sunday then the post office is closed.

Therefore the post office is closed.

(a) Translate the two statements into logic using propositions p, q and logical operations. Say what your p and q are.

(b) Identify the premises and the conclusion.

(c) What is the name for this type of argument?

(8) Suppose these two statements are true:

- If it is Sunday then the post office is closed.
- If the post office is closed then I cannot mail my letter.

(a) What can you conclude?

(b) What is the name for this type of argument?

(9) Give your own example in words of a modus tollens argument. Write the premises clearly and give the conclusion.

(10) Here are four propositions:

- b says "the cat hides behind a bush"
- t says "the cat climbs a tree"
- s says "the dog sees the cat"
- c says "the dog chases the cat".

Suppose these premises are true

$$b \rightarrow \neg s$$
$$s \rightarrow c$$
$$c \rightarrow t$$
$$\neg t$$

(a) Write these four premises in simple English.

(b) Can we conclude that $\neg s$ is true? Explain.

(c) Can we conclude that b is true? Explain.

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.

- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).

- Ask me about it after class.

- Come to my office hours: Tue 3 - 4, Wed 3 - 4 in CP 317.

- Go to the Math Tutorial Lab in-person in CP 303 or online.