CSI 30, Homework 2 on section 1.2, 1.3, 1.4

Due by Wed, Feb 22.

Write all your working out and answers on your own notepaper. Please use lots of space and as many pages as you want, so I can include corrections or comments - otherwise I will ask you to redo it. You do not need to write the questions, but it is very important that you explain your answers and show clearly any work you had to do. Each question is worth 3 points for a total of 30.

---

(1) Are these system specifications consistent?

- Whenever the system software is being upgraded, users cannot access the file system.
- If users can access the file system, then they can save new files.
- If users cannot save new files, then the system software is not being upgraded.

(Translate these into 3 compound propositions and use a truth table to see if they can all be true together.)

(2) On the island of Knights and Knaves, the Knights only say true things and the Knaves only say false things. You meet two people A and B.

A says "B is a knave"
B says "We're both knights"

Decide if A and B are knights or knaves and explain you answer.

(3) Is $\neg(p \to p)$ a tautology, a contradiction or a contingency. Explain your answer.

(4) Use De Morgan's laws to find the negation of this statement:

I will wash the dishes and take out the trash.

(5) Use a truth table to prove that $\neg(p \oplus q) \equiv p \leftrightarrow q$

(6) Use laws of logic (and not a truth table) to prove that

$$p \vee \neg(p \wedge q)$$

is a tautology (always true). Justify each of your steps.

(7) The propositional function $P(x)$ says "$x = x^2$". Decide the truth values of:

(a) $P(-1)$

(b) $P(0)$

(c) $P(1)$

(d) $P(5)$

(The answer to each part is true or false.)

(8) Let $R(x)$ be the statement "the word x contains the letter a". Give the truth values of these:

(a) $R(\text{orange})$

(b) $R(\text{lemon})$

(c) $R(\text{true})$

(d) $R(\text{false})$

(9) Suppose $Q(x)$ says "x can speak Swedish". Let the domain be all people living in New York City.

(a) Explain in words what $\forall x\, Q(x)$ means.

(b) Is $\forall x\, Q(x)$ true?

(c) Explain in words what $\exists x\, Q(x)$ means.

(d) Is $\exists x\, Q(x)$ true?

(10) As before, let $P(x)$ say "$x = x^2$". Let the domain be the set of integers. Decide the truth values of these, explaining your answer:

(a) $\forall x\, P(x)$

(b) $\exists x\, P(x)$

(c) $\exists! x\, P(x)$

(d) $\exists x\, \neg P(x)$

If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Tue 3 - 4, Wed 3 - 4 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.