

CSI 30, Homework 10 on sections 6.3, 6.4, 7.1

Due by Wed, May 10.

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Please use lots of space and explain your answers, showing clearly any work you had to do. Each question is worth 3 points for a total of 33.

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(1) How many permutations of 2 elements from the set  $\{a, b, c, d, e\}$  are there? List them all.

(Hint: there should be  $P(5, 2)$  permutations in your list.)

(2) Compute these permutation and combination numbers, showing your work:

(a)  $P(6, 3)$

(b)  $P(5, 5)$

(c)  $C(5, 5)$

(d)  $C(8, 4)$

(e)  $C(6, 0)$

(Hint: use the formulas

$$P(n, r) = \frac{n!}{(n-r)!}, \quad C(n, r) = \frac{n!}{(n-r)!r!}$$

and there is always lots of cancellation.)

(3) Suppose there is a group of 8 people.

(a) How many different teams of 4 can be chosen from this group?

(b) How many ways can a photographer select and arrange 4 people from this group to stand in a line?

(4) (a) How many subsets of size 3 does a set with 20 elements have?

(b) How many bit strings of length 20 have exactly 17 ones?

(5) Use the binomial theorem to expand  $(x + 3)^6$

(6) If we have the numbers

1 8 28 56 70 56 28 8 1

on a row of Pascal's triangle, use Pascal's identity to find the next row.

(7) Work out these three questions directly or by using the binomial theorem. (It's a lot less work if you use the binomial theorem!)

(a) Find:  $\binom{8}{0} - \binom{8}{1} + \binom{8}{2} - \binom{8}{3} + \binom{8}{4} - \binom{8}{5} + \binom{8}{6} - \binom{8}{7} + \binom{8}{8}$

(b) Find:  $\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8}$

(c) Find:  $\binom{8}{0} + 2\binom{8}{1} + 4\binom{8}{2} + 8\binom{8}{3} + 16\binom{8}{4} + 32\binom{8}{5} + 64\binom{8}{6} + 128\binom{8}{7} + 256\binom{8}{8}$

(8) You pick a random card from a standard deck. Find the probability

- (a) that it is the ace of clubs,
- (b) that it is a king,
- (c) that it is a ten or a club.
- (d) Which of (a), (b), (c) is least likely to happen?

(9) Roll two dice.

- (a) Find the probability that their sum is 8.
- (b) Find the probability that their sum is not 8.

(10) Show that the probability of a 5-card poker hand containing exactly one heart is 41.1%.

- (11) (a) A lottery uses the numbers 1 to 51 and picks 6 of these at random. You must match 6 numbers, in any order, to win. Find the probability of your six chosen numbers matching and you winning the lottery.
- (b) If you toss a coin 24 times, what is the probability of getting tails every time?
- (c) Which of (a), (b) is most likely to happen?

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If you get stuck on a question or aren't sure if you understand it:

- Go over the relevant class notes and section in the textbook.
- Check if you get the right answer for a similar odd-numbered question in the textbook (answers at the back of the book).
- Ask me about it after class.
- Come to my office hours: Tue 3 - 4, Wed 3 - 4 in CP 317.
- Go to the Math Tutorial Lab in-person in CP 303 or online.