Math 30, Practice Final Exam

This practice final is to give you an idea of what to expect and help us review. Any questions that are similar to quiz and homework questions could appear on the final.

On the final you will be asked to do all questions in 2 hours and 50 minutes. They are each worth different numbers of points, depending on how long they are, and it is very important that you show clearly all your working out and reasoning. You may use a basic scientific calculator. Phones must be put away.

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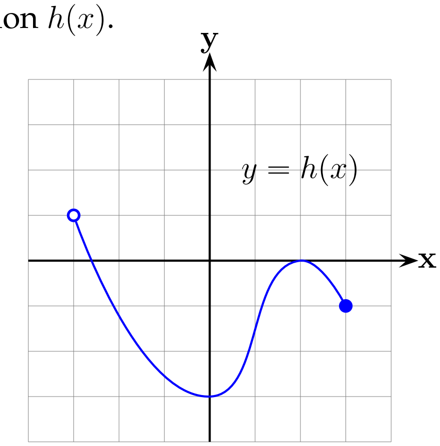
(1) This is the graph of the function $h(x)$.



Find the following:

(a) $h(3)$

(b) the domain of h

(c) the range of h

(d) $(h \circ h)(2)$

(2) For the functions $f(x) = 3x$ and $g(x) = x^2 - x + 2$, compute and simplify

(a) $(f - g)(x)$

(b) $(fg)(1)$

(c) $(f \circ g)(x)$

(d) $(g \circ f)(-1)$

(3) Let $f(x) = 4x + 1$ and $g(x) = \dfrac{x-1}{4}$.

(a) Compute $(f \circ g)(x)$.

(b) Compute $(g \circ f)(x)$.

(c) Say why these functions are inverses of each other (or why not).

(4) Use synthetic division and the Remainder Theorem to find $f(-2)$ for the polynomial

$$f(x) = 3x^3 - 6x^2 + 2x - 7.$$

(5) Let

$$f(x) = x^3 + 4x^2 - 4x - 16.$$

(a) List all possible rational zeros of $f(x)$ according to the Rational Zeros Theorem.

(b) Find one actual zero by using synthetic division to test the possible zeros from (a).

(c) Find all remaining zeros by factoring the quotient from the zero you found in (b).

(d) Write down the complete factorization of $f(x)$.

(e) Sketch the graph of $f(x)$, making sure you show all x and y intercepts.

(6) Find the x-intercepts and vertical asymptotes of the graph of the rational function

$$g(x) = \frac{(x+2)(x+4)}{(x+1)(x-2)}.$$

Find the y intercept and horizontal asymptote. Plot some extra points and then sketch the graph. Remember to label and number the axes.

(7) Solve and give the solutions in interval notation

$$\frac{(x+1)(x+5)}{(x+2)(x-3)} < 0.$$

(8) Evaluate these logarithms:

(a) $\log_5 125$

(b) $\log_{17} 289$

(c) $\log_{49} 7$

(d) $\log_4(1/64)$

(e) Express $\log_2 1000$ using the common logarithm (base 10). Then use your calculator to evaluate it correct to 4 decimal places.

(9) If $\log_b x = 12$ and $\log_b y = 4$ then find:

(a) $\log_b(xy)$

(b) $\log_b(x/y)$

(c) $\log_x b$

(10) Solve exactly:

(a) $7^{x+3} = 8^x$

(b) $\log_7 x + \log_7(x + 48) = 2$

(11) Let

$$g(x) = \log_4(2x + 9)$$

and find $g^{-1}(x)$.

(12) Find exactly:

(a) $\sin(8\pi/3)$

(b) $\cos^{-1}(\sqrt{2}/2)$

(c) $\sin(\cos^{-1}(6/7))$

(13) Determine the amplitude, period and phase shift of $y = 3\sin(2x - \pi/2)$. Then graph one period of the function by first plotting the 5 key points.

(14) Verify the trigonometric identities:

(a)

$$\frac{\sin\theta}{1+\cos\theta} + \frac{-1+\cos\theta}{\sin\theta} = 0$$

(b)

$$\sqrt{2}\sin(x + \pi/4) = \sin(x) + \cos(x)$$

(15) Solve:

(a) $\sin(3x) = 1$ for x in $[0, 2\pi)$

(b) $\sqrt{2}\cos(x) = -1$ for x in $[0, 2\pi)$

Formulas

$$\begin{aligned}
\sin(\alpha + \beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)\\
\sin(\alpha - \beta) &= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)\\
\cos(\alpha + \beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)\\
\cos(\alpha - \beta) &= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)
\end{aligned}$$